

On Modeling the Magnetoelectric Properties of a Myelinated Axon using the Finite Element Method

Ralf Rumbler¹, Bojana Petković¹, Marek Ziolkowski², Jens Haueisen², Žaklina Mančić³, Zlata Cvetković³, Thomas Jochmann², and Hannes Toepfer¹

¹Advanced Electromagnetics Group, Technische Universität Ilmenau, Ilmenau 98693, Germany, ralf.rumbler@tu-ilmenau.de, bojana.petkovic@tu-ilmenau.de, hannes.toepfer@tu-ilmenau.de

²Institute of Biomedical Engineering and Informatics, Technische Universität Ilmenau, Ilmenau 98693, Germany, jens.haueisen@tu-ilmenau.de, thomas.jochmann@tu-ilmenau.de

³Faculty of Electronic Engineering, University of Niš, Aleksandra Medvedeva 14, 18000 Niš, Serbia, zaklina.mancic@elfak.ni.ac.rs, zlata.cvetkovic@elfak.ni.ac.rs

We present a finite element modeling of magnetoelectric properties of a myelinated axon. Results are compared against an analytical solution for the case of an axon exposed to a homogeneous electric field. We achieve a mean relative error below 0.3% and a normalized root mean square deviation below 0.5%.

Index Terms—Analytical models, Axons, Finite element methods, Magnetoelectric effects, Myelin.

I. INTRODUCTION

IN THE HUMAN NERVOUS SYSTEM, axon fibers connect the bodies of nerve cells and enable electrical signaling. An isolating sheath called *myelin* surrounds the axons to facilitate rapid transmission of action potentials [1] and to provide metabolic support [2]. Damages to the myelin sheath occur in various neurological disorders and lead to loss of axonal function [3]. Myelinated tissue constitutes approximately 25% of the brain, substantially influencing its electromagnetic properties, which are crucial for diagnostic and therapeutic applications. Consequently, evaluating myelin under both normal and pathological conditions is critical for clinical diagnosis and neurophysiological research.

The myelin sheath is characterized by its dielectric permittivity [4] and magnetic susceptibility [5]. The influence of the magnetic susceptibility in myelin on the perturbation of the magnetic field has been studied in [6]. The transmembrane potential of the sheath-covered axonal segment under the transverse electric field stimulation accounts for a dielectric permittivity of myelin [4]. However, recent research shows that biological cells act as soft magnetoelectric materials [7]. These materials have the ability to magnetize under the application of an electric field and electrically polarize under the application of a magnetic field. Specifically in neuronal cells, a coupling of the magnetic and electric fields may pose significant consequences to processes like magnetoreception. We expand the existing models of myelinated axons by coupling electric and magnetic fields in a myelin sheath. We calculate the induced magnetic flux density vector inside the myelination, intra-axonal and extra-axonal medium, exposed to a transverse electric field.

The model description is given in section II. Finite element and analytical solutions are presented in sections III and IV, respectively. Numerical results and discussion are given in section V.

II. MODEL DESCRIPTION

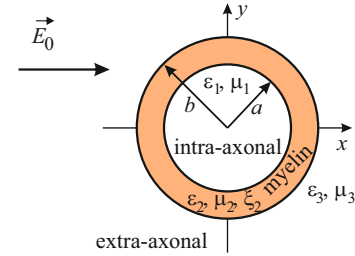


Fig. 1. Magnetoelectric model of a myelinated axon exposed to an external homogeneous electric field $\vec{E}_0$.

We model the myelinated axon as a two-layer cylinder with an inner and outer radius of a myelin sheath equal to $a = 0.3\mu\text{m}$ and $b = 0.6\mu\text{m}$ [8], respectively (Fig. 1). For the intra- and extra-axonal media we assume a permittivity of $\epsilon_1, \epsilon_3 = 6.4 \cdot 10^{-10} \text{ As/Vm}$ [4] and a permeability of free space $\mu_1, \mu_3 = \mu_0 = 4\pi \cdot 10^{-7} \text{ H/m}$ [5]. We model the myelin sheath with a permittivity $\epsilon_2 = 4.4 \cdot 10^{-11} \text{ As/Vm}$ [4] and a magnetic susceptibility of $\chi = -10^{-5}$ [5]. The magnetoelectric coupling factor is $\xi_2 = \sqrt{p \epsilon_2 \mu_2}$, where the level of bi-isotropic effect is set to $p = 0.2$. The axon is exposed to a homogeneous electric field $E_0 = 200 \text{ V/m}$ in x -direction. The myelin sheath is considered as Tellegen medium [9]:

$$\vec{B}_2 = \mu_2 \vec{H}_2 + \xi_2 \vec{E}_2 \quad (1)$$

$$\vec{D}_2 = \epsilon_2 \vec{E}_2 + \xi_2 \vec{H}_2 \quad (2)$$

III. FINITE ELEMENT SOLUTION

We provide a finite element solution of the electric and magnetic field distributions in the myelin sheath and intra- and extra-axonal medium. We used COMSOL Multiphysics® (v6.2) [10]. The model is realized using the modules "Electrostatics" and "Magnetic Fields, No Currents" simultaneously. The non-magnetoelectric material properties in medium 1 and 3 are

configured using the constant permittivity and permeability values under the "Constitute Relation E-D" and "Constitute Relation B-H", respectively.

The magnetoelectric properties of the myelin sheat are modeled through "Remanent flux density" and "Remanent electric displacement". The permeability μ_2 is added as "Recoil permeability μ_{rec} ". For the "Remanent flux density norm" we added " $xi * es.normE$ ", where " xi " stands for the value of ξ and " $es.normE$ " for the norm of the electric field vector. The "Remanent flux direction" is specified in the direction of the electric field vector by entering " $es.Ex/1[V/m]$ " and " $es.Ey/1[V/m]$ " to the respective columns. These expressions return each component of the electric field vector without its unit. For establishing equation (2) we supplied " $xi*mfnc.Hx$ " and " $xi*mfnc.Hy$ " for the "Remanent electric displacement", with the latter factors representing the components of the magnetic field.

IV. REFERENCE ANALYTICAL SOLUTION

To validate the numerical results, we solve the Laplace equation analytically using the method of separation of variables. Electric and magnetic scalar potentials are represented by the following expressions:

$$\begin{aligned} \varphi_e &= \cos(\alpha)Cr, \varphi_m = \cos(\alpha)Kr, r \leq a \\ \varphi_e &= \cos(\alpha)(Dr + F/r), \varphi_m = \cos(\alpha)(Lr + M/r), a \leq r \leq b \\ \varphi_e &= \cos(\alpha)(-E_0r + G/r), \varphi_m = \cos(\alpha)(N/r), b \leq r \end{aligned} \quad (3)$$

where $r = \sqrt{x^2 + y^2}$ and $\alpha = \text{atan2}(y, x)$. The constants will be given in the full version of the paper. Expressions (1) and (2) yield the vectors $\vec{B}$ and $\vec{D}$.

V. RESULTS AND DISCUSSION

The mean relative error (RE) with standard deviation (SD) and the normalized root mean square deviation ($NRMSE$) for the norms of the vectors $\vec{E}$ and $\vec{B}$ are represented in Table I. The mean RE is below 0.3% and a $NRMSE$ below 0.5%. Due to magnetoelectric properties of a myelin sheath, the magnetic flux density occurs applying an external electric field. The distribution of the norm of a magnetic flux density vector is represented in Fig. 2. The field norm is constant in the intra-axonal medium. The norm of the electric field along the x -axis is represented in Fig. 3. It is largest in the myelin sheath.

TABLE I
MEAN RE [%] WITH SD AND $NRMSE$ [%] FOR THE NORMS $\|\vec{E}\|$ AND $\|\vec{B}\|$.

	Mean RE [%] (SD)		$NRMSE$ [%]	
	$\ \vec{E}\ $	$\ \vec{B}\ $	$\ \vec{E}\ $	$\ \vec{B}\ $
Intra-axonal	0.0551 (0.0177)	0.1368 (0.0080)	-	-
Myelin sheath	0.1936 (0.3141)	0.1196 (0.0393)	0.1496	0.4755
Extra-axonal	0.0910 (0.1452)	0.2517 (0.2182)	0.0789	0.1650

VI. CONCLUSION

Finite element model can account for magnetoelectric properties of a myelinated axon. In the full paper we will expand the model to use anisotropic electromagnetic quantities for the myelin sheath [11], to account for its layered structure.

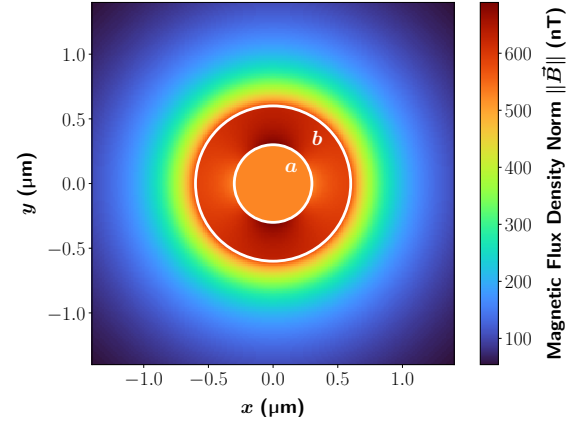


Fig. 2. Distribution of the norm of magnetic flux density vector in a myelin sheath and intra- and extra-cellular medium.

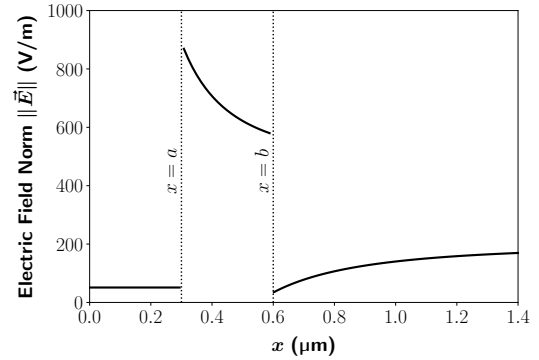


Fig. 3. Distribution of the norm of electric field vector along the x -axis in a myelin sheath and intra- and extra-cellular medium.

REFERENCES

- [1] J. W. Moore, R. W. Joyner, M. H. Brill, S. D. Waxman, M. Najar-Joa, "Simulations of conduction in uniform myelinated fibers. Relative sensitivity to changes in nodal and internodal parameters," *Biophysical Journal*, vol. 21, pp. 147–160, 1978.
- [2] A. Hasenstaub, S. Otte, E. Callaway, T.J. Sejnowski, "Metabolic cost as a unifying principle governing neuronal biophysics," *Proc. Natl. Acad. Sci. U.S.A.*, vol. 107 (27), pp. 12329–12334, 2010.
- [3] K.-A. Nave, "Myelination and Support of Axonal Integrity by Glia," *Nature*, vol. 468 (7321), pp. 244–252, 2010.
- [4] Y. Hui, and J. Ng, "Shielding effects of myelin sheath on axolemma depolarization under transverse electric field stimulation," *PeerJ*, vol. 6 e6020, Dec. 2018.
- [5] T. Xu, S. Foxley, M. Kleinnijenhuis, W.C. Chen and K.L. Miller, "The effect of realistic geometries on the susceptibility-weighted MR signal in white matter," *Magn Reson Med.*, vol. 79(1), pp. 489–500, Apr. 2018.
- [6] Roth BJ, Puwal S, Basser PJ. "Local magnetic field perturbations caused by magnetic susceptibility heterogeneity in myelin-water layers within an axon," *Journal of Coupled Systems and Multiscale Dynamics* vol. 3(3), pp. 228–232, 2015.
- [7] S. Krichen, L. Liu and P. Sharma, "Biological cell as a soft magnetoelectric material: Elucidating the physical mechanisms underpinning the detection of magnetic fields by animals," *Phys. Rev. E*, vol. 96, 2017.
- [8] Y. Bakiri, R. Káradóttir, L. Cossell, and D. Attwell, "Morphological and electrical properties of oligodendrocytes in the white matter of the corpus callosum and cerebellum," *J Physiol.*, vol. 589.3, pp. 559–573, Feb. 2011.
- [9] A. H. Sihvola, "Electromagnetic modeling of bi-isotropic media," *Progress in Electromagnetic Research - PIER*, vol. 9, pp. 45–86, 1994.
- [10] *COMSOL multiphysics*, (2023). COMSOL AB. Stockholm, Sweden.
- [11] S. Puwal, B. J. Roth, and P.J. Basser, "Heterogeneous anisotropic magnetic susceptibility of the myelin-water layers causes local magnetic field perturbations in axons," *NMR Biomed.*, vol. 30(4), Apr. 2017.