

Fast MOR-Based Approach to Transcranial Magnetic Stimulation for Varying Excitation Coil Position

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A nonlinear parametric Model Order Reduction approach is proposed and applied for the computation of the induced electric field in a subject's head due to Transcranial Magnetic Stimulation. The resulting reduced order model dramatically decreases computational times when simulating several positions and orientations of the excitation coil in a chosen region of interest. In particular the proposed model allows to obtain field solutions in a faster way with respect to classical solvers as Finite Element Methods without losing accuracy.

Index Terms—Transcranial magnetic stimulation, Model order reduction, induced electric field

I. INTRODUCTION

Transcranial Magnetic Stimulation (TMS) is a well-known method for stimulating the cortex of the brain with an electric current density induced by an external time-varying magnetic field. In medical practice, a coil, excited by a time-varying current, is positioned on the patient's skull contactless so that the generated magnetic field induces an electric field within the brain tissues, which, in turn, is responsible for neuronal stimulation. The main drawback of this technique is the lack of precision in stimulating a target point because of the spread of the induced electric field: an accurate computation of this field inside the brain, specifically in the gray matter, is of paramount importance [1] and several studies investigated this issue [2].

The fast nonlinear parametric Model Order Reduction (MOR) approach proposed in this paper can be extremely beneficial in optimization studies and real time applications. Unlike the linear parametric MOR approach in [3], in which the variation of the tissue electrical parameters is considered, here the parameters of interest are those describing the position and orientation of the excitation coil.

II. MODEL ORDER REDUCTION BASED APPROACH

We consider a subject's head Ω and a figure-of-eight excitation coil centered at a chosen point $\mathbf{t}$ of a surface Σ placed at a distance δ above the scalp and covering a chosen cortex area. The coil can be rotated around the axis normal to Σ passing through $\mathbf{t}$ by the rotation matrix $\mathbf{R}$.

Due to the small values of electric conductivity in the head and of excitation coil angular frequency ω , the electromagnetic problem can be simplified as a time-harmonic eddy current problem in which the secondary magnetic field from the induced eddy currents is neglected. As a result the magnetic induction $\mathbf{b}(\mathbf{r})$, function of position vector $\mathbf{r}$, is given by $\nabla \times \mathbf{a}_c(\mathbf{r} | \mathbf{R}, \mathbf{t})$, being $\mathbf{a}_c(\mathbf{r} | \mathbf{R}, \mathbf{t})$ the vector potential for the

coil rototranslated by the rotation matrix $\mathbf{R}$ and the translation vector $\mathbf{t}$. Continuity equation for the current density $\mathbf{j}(\mathbf{r})$ reads

$$\nabla \cdot \mathbf{j}(\mathbf{r}) = 0. \quad (1)$$

Ohm's law relates $\mathbf{j}(\mathbf{r})$ to the electric field $\mathbf{e}(\mathbf{r})$ by means of the scalar electric conductivity $\sigma(\mathbf{r})$

$$\mathbf{j}(\mathbf{r}) = \sigma(\mathbf{r})\mathbf{e}(\mathbf{r}). \quad (2)$$

Faraday's equation written in terms of scalar potential $\varphi(\mathbf{r})$ and vector potential $\mathbf{a}_c(\mathbf{r} | \mathbf{R}, \mathbf{t})$ holds

$$\mathbf{e}(\mathbf{r}) = -\nabla\varphi(\mathbf{r}) - i\omega\mathbf{a}_c(\mathbf{r} | \mathbf{R}, \mathbf{t}). \quad (3)$$

No electric current goes through the contour $\partial\Omega$ of the head,

$$\mathbf{j}(\mathbf{r}) \cdot \mathbf{n}(\mathbf{r}) = 0, \quad (4)$$

$\mathbf{n}(\mathbf{r})$ being the outward unit vector normal to $\partial\Omega$.

A tetrahedral mesh of the head is obtained with SimNIBS v. 4.0 [4] starting from a freely available Magnetic Resonance image of a subject ("Ernie" case study). The TMS problem can then be discretized over the obtained mesh having N nodes and E edges, using N linear nodal finite elements for $\varphi(\mathbf{r})$ and E linear edge finite elements for $\mathbf{a}_c(\mathbf{r} | \mathbf{R}, \mathbf{t})$, in the form

$$\mathbf{K}\varphi = -i\omega\mathbf{F}\mathbf{a}_c(\mathbf{R}, \mathbf{t}), \quad (5)$$

in which φ is the N -rows column vector with the Degrees of Freedom (DoF) of $\varphi(\mathbf{r})$, and $\mathbf{a}_c(\mathbf{R}, \mathbf{t})$ is the E -rows column vector with the DoF of $\mathbf{a}_c(\mathbf{r} | \mathbf{R}, \mathbf{t})$. Besides, $\mathbf{K} = \mathbf{G}^T \mathbf{M}_\sigma \mathbf{G}$ and $\mathbf{F} = \mathbf{G}^T \mathbf{M}_\sigma$, in which $\mathbf{G}$ is the $E \times N$ edge-node incidence matrix and $\mathbf{M}_\sigma$ is the E -order discrete constitutive matrix.

Algorithm 1 allows to build the $N \times \hat{n}$ matrix $\mathbf{V}$ and the $E \times \hat{n}$ matrix $\mathbf{Q}$ allowing the extraction of a parametric Reduced Order Model (ROM). Precisely, the $\hat{n} \ll N$ columns of matrix

$\mathbf{V}$ span the solution φ of (5) for all admissible values of $\mathbf{R}$ and $\mathbf{t}$ so that

$$\varphi = \mathbf{V}\hat{\mathbf{x}}, \quad (6)$$

being $\hat{\mathbf{x}}$ a column vector of $\hat{n}$ DoFs. By applying Galerkin's projection to (5) it ensues

$$\hat{\mathbf{K}}\hat{\mathbf{x}} = -i\omega \mathbf{V}^T \mathbf{F} \mathbf{a}_c(\mathbf{R}, \mathbf{t}) \quad (7)$$

being $\hat{\mathbf{K}} = \mathbf{V}^T \mathbf{K} \mathbf{V}$. An hyper-reduction step is then performed as follows. The $\hat{n}$ columns of matrix $\mathbf{Q}$ span vector $\mathbf{a}_c(\mathbf{R}, \mathbf{t})$ for all admissible values of $\mathbf{R}$ and $\mathbf{t}$. Thus exploiting [5], an $E \times \hat{e}$ row selection matrix $\mathbf{S}$ is derived from $\mathbf{Q}$, with $\hat{e} \ll E$, from which the parametric ROM model ensues in the form

$$\hat{\mathbf{K}}\hat{\mathbf{x}} = -i\omega \hat{\mathbf{F}} \hat{\mathbf{a}}_c(\mathbf{R}, \mathbf{t}) \quad (8)$$

in which

$$\hat{\mathbf{F}} = \mathbf{V}^T \mathbf{F} \mathbf{Q} \hat{\mathbf{Q}}^+ \quad (9)$$

being the $\hat{n} \times \hat{e}$ reduced order matrix $\hat{\mathbf{Q}}^+$ the pseudoinverse of the sub-matrix $\hat{\mathbf{Q}} = \mathbf{S}^T \mathbf{Q}$ of $\mathbf{Q}$ extracted from the $\hat{e}$ selected rows and similarly

$$\hat{\mathbf{a}}_c(\mathbf{R}, \mathbf{t}) = \mathbf{S}^T \mathbf{a}_c(\mathbf{R}, \mathbf{t}) \quad (10)$$

is the sub-vector of $\mathbf{a}_c(\mathbf{R}, \mathbf{t})$ extracted from the $\hat{e}$ selected rows.

Algorithm 1: Matrices $\mathbf{V}$ and $\mathbf{Q}$

```

Set  $\mathbf{V} := []$ ; Set  $\mathbf{Q} := []$ 
Set flag := TRUE
while flag do
    Set steps := 0
    while steps < STOP do
        Pick random  $\mathbf{R}, \mathbf{t}$ 
        Compute  $\mathbf{a}_c(\mathbf{R}, \mathbf{t}), \boldsymbol{\rho} = \mathbf{F} \mathbf{a}_c(\mathbf{R}, \mathbf{t})$ 
        if  $\mathbf{V} = []$  then
            break
        Set steps := steps + 1
        Solve  $\hat{\mathbf{K}}\hat{\mathbf{x}} = \mathbf{V}^T \boldsymbol{\rho}$  for  $\hat{\mathbf{x}}$ 
        if  $\|\boldsymbol{\rho} - \mathbf{K} \mathbf{V} \hat{\mathbf{x}}\|_2 > \text{TOL} \|\boldsymbol{\rho}\|_2$  then
            Set flag := TRUE
            break
        else
            Set flag := FALSE
    if flag then
        Solve  $\mathbf{K}\boldsymbol{\varphi} = \boldsymbol{\rho}$  for  $\boldsymbol{\varphi}$ 
        Set  $\mathbf{V} := [\mathbf{V} \boldsymbol{\varphi}]$ ; Set  $\mathbf{Q} := [\mathbf{Q} \mathbf{a}_c(\mathbf{R}, \mathbf{t})]$ 
        Set  $\hat{\mathbf{K}} := \mathbf{V}^T \mathbf{K} \mathbf{V}$ 

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III. NUMERICAL RESULTS

The FEM model of a subject's head can be built and solved in about 30s using an in-house MATLAB code run on a 2.4Ghz 8-Core Intel Core i9 over a tetrahedral mesh of about $V = 4.4$

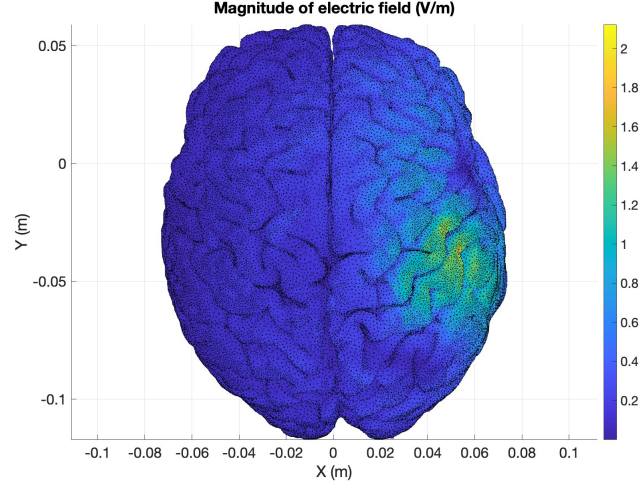


Fig. 1. Electric field magnitude on the surface of the white matter body.

million volumes and having about $N = 0.8$ million nodes and $E = 5.3$ million edges. The coil is a Magstim 70mm carrying a 1A current at $\omega = 1,000$ rad/s, which can be centered at any point of a surface Σ having a diameter of 3 cm, positioned at a distance $\delta = 0.5$ cm from the cortex area around a given point, as typical in clinical practice.

Using Algorithm 1, a parametric ROM with $\hat{n} = 58$ and $\hat{e} = 116$ is extracted in about 3.6 min. This parametric ROM can be solved in less than 1.5ms, with a speedup larger than 20,000, and is able to reconstruct the electric field due to TMS, shown in Fig. 1, with a relative error lower than 0.5% with respect to the solution of the FEM model, for all choices of the coil position over Σ .

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