

Theoretical Analysis of Cogging Torque Minimization in Dual-Stator Single-Rotor Axial Flux Permanent Magnet Machines

Xiang Cao¹, Fei Zhao¹, Dandan Wang¹, Ruixiang Liu¹, Chao Zhi², and Haotian Wu³

¹School of Robotics and Advanced Manufacturing, Harbin Institute of Technology (Shenzhen), Shenzhen, 518055 China

²Meituan Technology, Huilong Business Center, Longhua District, Shenzhen, 518100 China

³Meituan Technology, Taiji Building, Chaoyang District, Beijing, 100102 China

Axial flux permanent magnet (AFPM) machines are well-suited for industrial manufacturing, electric transportation and a variety of direct drive and low speed applications, due to their advantages in high torque density, high power density, and axial compactness. Since cogging torque can exacerbate torque ripple, vibration, and noise in these applications, a dual air-gap phase shift scheme is proposed in this paper to minimize the cogging torque of dual-stator single-rotor (DSSR) AFPM machines based on the theoretical calculation and analysis. Three specific methods, permanent magnets (PMs) shift, stators shift, and tooth tips shift, are introduced and validated through finite element analysis (FEA), where significant cogging torque reduction is demonstrated.

Index Terms—Cogging torque minimization, phase shift, axial flux permanent magnet (AFPM), dual-stator single-rotor (DSSR).

I. INTRODUCTION

Axial flux permanent magnet (AFPM) machines, characterized by high torque density, high power density, and axial compactness, are widely applied in industrial traction, electric vehicles, wind generators, and a variety of direct drive as well as low speed applications[1]. In these applications, strict requirements are imposed on the quality of the output torque, with torque ripple being considered a key factor in evaluating torque quality[2]. Minimizing cogging torque is critical in machine systems, as it not only exacerbates torque ripple, but also generates noise and vibration, which can severely impact machine performance, particularly in low speed and direct drive applications[3].

In this paper, three methods are proposed to minimize the cogging torque of a dual-stator single-rotor (DSSR) AFPM machine, based on theoretical calculation and analysis of cogging torque in AFPM machines. The effectiveness of these methods is validated using the finite element analysis (FEA), and the results demonstrate that all three methods can significantly reduce cogging torque and torque ripple.

II. COGGING TORQUE MINIMIZATION METHODS

Based on the energy method, the cogging torque of an AFPM machine is expressed as

$$T_{cog}(\theta_p) = K \sum_{n=1}^{\infty} \frac{1}{n} \int_{R_i}^{R_o} r \sin\left(nN_L \frac{b_1}{2r}\right) dr \times \sin\left(nN_L \frac{\alpha_p \pi}{N_p}\right) \sin(nN_L(\theta_p + \theta_x)) \quad (1)$$

$$K = -\frac{2gB_r^2 N_s N_p}{\mu_0 \pi N_L} \quad (2)$$

where B_r is the maximum magnetic flux density in the air gap of slotless machine, g is the axial length of the air gap, N_s is the number of stator slots, N_p is the number of rotor poles, $N_L = \text{LCM}(N_s, N_p)$ is the number of cogging torque cycles per machine revolution, R_o is the outer radius of the machine, R_i is

the inner radius of the machine, r is the radius of the machine, b_1 is the slot opening width expressed in length at the radius r , α_p is the pole arc coefficient of machine, θ_p is the position angle of the rotor, and θ_x is the relative shift angle between the stator and rotor when θ_p is determined. Taking the initial position of a single-stator single-rotor (SSSR) AFPM machine as an example, at this moment $\theta_p = 0$. If $\theta_x = 0$, then the center of slot I aligns with the center of permanent magnet (PM) I. θ_x represents the relative shift angle between the centers of slot I and PM I, as shown in Fig. 1(a) for the case where $\theta_x \neq 0$.

The fundamental wave and harmonics of the cogging torque are phase-shifted versus θ_x when other parameters are fixed. In a DSSR AFPM machine, the total cogging torque is the result of the superposition of the cogging torques in two air gaps, and the cogging torque in each air gap can be independently phase-shifted. The structure of a 24-slot, 20-pole machine is shown in Fig. 1(b). Defining the air gaps close to stator I and II as air gap I and II, respectively. If the cogging torque phase shift on the air gap I side is θ_x , and the cogging torque phase shift on the air gap II side is $-\theta_x$, resulting in a total phase difference of $2\theta_x$, then the cogging torque formula becomes

$$T_{totalcog}(\theta_p) = K \sum_{n=1}^{\infty} \frac{1}{n} \int_{R_i}^{R_o} r \sin\left(nN_L \frac{b_1}{2r}\right) \sin\left(nN_L \frac{\alpha_p \pi}{N_p}\right) dr \times \left(\sin(nN_L(\theta_p + \theta_x)) + \sin(nN_L(\theta_p - \theta_x))\right) \quad (3)$$

at this time, as long as

$$\sin(nN_L(\theta_p + \theta_x)) + \sin(nN_L(\theta_p - \theta_x)) = 0 \quad (4)$$

that is

$$nN_L \theta_x = \frac{2k-1}{2} \pi, \quad k = 1, 2, 3, \dots \quad (5)$$

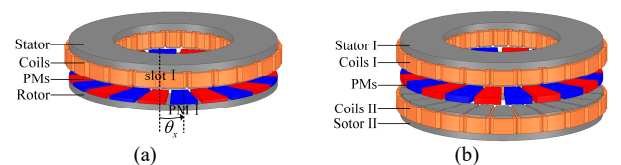


Fig. 1. Machine structure. (a) SSSR AFPM. (b) DSSR AFPM.

then, $T_{totalcog}(\theta_p) = 0$. As can be seen from Fig. 3(b), the fundamental wave of the cogging torque of the original machine occupies a significant proportion. Taking $N_s = 24$, $N_p = 20$, $n = 1$ and $k = 1$, the minimum shift angle $\theta_x = 0.75^\circ$ is obtained to make the fundamental wave amplitude 0. At this point, the amplitudes of the odd harmonics are also 0, while the amplitudes of the even harmonics remain unchanged. As a result, the cogging torque can be significantly reduced.

For DSSR AFPM machines, the cogging torque phase can be shifted through PMs shift, stators shift, and tooth tips shift, as shown in Fig. 2. The specific methods are as follows: the PMs are divided into two groups, PMs I and PMs II, by a radial plane at the axial center, and are shifted by θ_x in the reverse rotational direction; the stator I and II are shifted by θ_x in the reverse rotational direction; the tooth tips I and II are shifted by θ_x in the reverse rotational direction.

III. FEA OF COGGING TORQUE MINIMIZATION METHODS

A. Cogging Torque Minimization Results

Fig. 3(a) presents a schematic representation of the theoretical fundamental waves of cogging torques for air gap I and II when $\theta_x = 0.75^\circ$, along with their superimposed effect. Fig. 3(b) compares the fast Fourier transform (FFT) analysis of the cogging torques for the original machine and the three improved machines when $\theta_x = 0.75^\circ$ in the FEA. The results present that the amplitudes of the fundamental wave and odd harmonics are significantly reduced by more than 60%, with a 91.1% reduction in the fundamental wave for the stators shift method, while the even harmonics amplitudes remain unchanged. The results are consistent with the theoretical analysis in II. The cogging torques of the three methods when $\theta_x = 0.75^\circ$ are shown in Fig. 3(c), where the pk-pk values of cogging torques are reduced by at least 70% compared to the original machines.

Since the corresponding waveforms are slightly affected in the PMs shift method and the tooth tips shift method, the optimal angles of the minimum cogging torque may deviate somewhat from theoretical predictions. Using θ_x as a variable for parameter scanning, the cogging torque results, which present that the optimal shift angles for achieving the minimum cogging torque are 0.875° , 0.75° , and 0.625° for the PMs shift, stators shift, and tooth tips shift methods respectively, are shown in Fig. 3(d). It can be seen that the impacts are slight, and the optimal angles are still obtained near 0.75° . The minimum cogging torques for each method are shown in the Fig. 3(e). The corresponding data mentioned above are listed in Table I.

B. Effect on Output Torque and Ripple

For all three methods, the shift angles at the minimum cogging torque are applied: $\theta_x = 0.875^\circ$ for the PMs shift, $\theta_x = 0.75^\circ$ for the stators shift, and $\theta_x = 0.625^\circ$ for the tooth tips shift. To simulate the improved machines under load, the same current and initial rotor position as those of the original machine are set. A comparison of the output torques is shown in Fig. 3(f) and Table II. It can be seen that the torque ripples of the three methods are sufficiently suppressed and their average output torques are essentially constant.

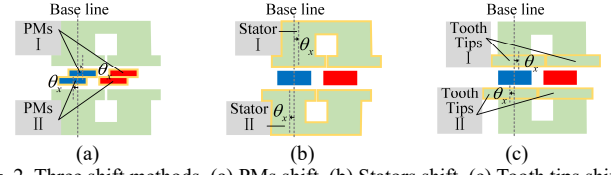


Fig. 2. Three shift methods. (a) PMs shift. (b) Stators shift. (c) Tooth tips shift.

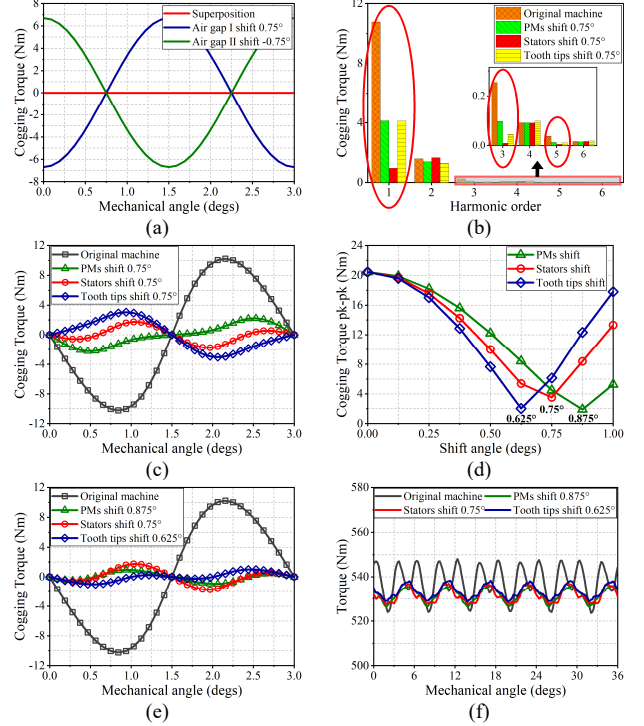


Fig. 3. Results. (a) Theoretical fundamental waves of cogging torques when $\theta_x = 0.75^\circ$. (b) FFT analysis of cogging torques when $\theta_x = 0.75^\circ$. (c) Cogging torques when $\theta_x = 0.75^\circ$. (d) Shift angle scanning of cogging torques. (e) Minimum cogging torques. (f) Output torques at minimum cogging torques.

TABLE I
COGGING TORQUE OF FOUR MACHINES

Machine	Shift angle	Cogging torque pk-pk (Nm)
Original Machine	-	20.45
PMs shift	0.75°	4.48 (-78.1%)
	0.875°	1.9 (-90.7%)
Stators shift	0.75°	3.52 (-82.8%)
	0.75°	6.13 (-70%)
Tooth tips shift	0.625°	2.03 (-90.1%)

TABLE II
TORQUE RIPPLE AND OUTPUT TORQUE AT MINIMUM COGGING TORQUE

Machine	Ripple	Output torque (Nm)
Original Machine	4.55%	537.75
PMs shift 0.875°	1.81% (-60.2%)	531.71 (-1.1%)
Stators shift 0.75°	1.75% (-61.5%)	531.71 (-1.1%)
Tooth tips shift 0.625°	1.8% (-60.4%)	533.28 (-0.8%)

REFERENCES

- [1] S. Kahourzade, A. Mahmoudi, H. W. Ping, and M. N. Uddin, "A comprehensive review of axial-flux permanent-magnet machines," *Can. J. Electr. Comput. Eng.*, vol. 37, no. 1, pp. 19-33, Winter 2014.
- [2] L. Xiao, J. Li, R. Qu, Y. Lu, R. Zhang, and D. Li, "Cogging torque analysis and minimization of axial flux PM machines with combined rectangle-shaped magnet," *IEEE Trans. Ind. Appl.*, vol. 53, no. 2, pp. 1018-1027, Mar./Apr. 2017.
- [3] L. Zhu, S. Z. Jiang, Z. Q. Zhu, and C. C. Chan, "Analytical methods for minimizing cogging torque in permanent-magnet machines," *IEEE Trans. Magn.*, vol. 45, no. 4, pp. 2023-2031, Apr. 2009.