

Unit Cell Modeling for Carbon Fiber Reinforced Polymers using an Integral Equation Method

Arnold Bingler^{1,2}, Tetsuya Uchimoto², Sándor Bilicz¹, and József Pávó¹

¹Budapest University of Technology and Economics, Faculty of Electrical Engineering and Informatics
Department of Broadband Infocommunications and Electromagnetic Theory, Budapest, 1103 Hungary
bingler.arnold@vik.bme.hu, bilicz.sandor@vik.bme.hu, pavo.jozsef@vik.bme.hu

²Institute of Fluid Science, Tohoku University, Sendai, Miyagi 980-8577, Japan, uchimoto@tohoku.ac.jp

In this paper, an integral equation method is proposed for the modeling of carbon fiber-reinforced polymers. To keep the number of degrees of freedom low, the fibers are modeled using a thin-wire approximation. The occasional contact points between the fibers are taken into account by lumped element resistors. The latter is incorporated into the discretized linear equation system by introducing zigzag basis functions, that enable the currents inside the fibers to jump at the contact points. This approach enables the analysis of a unit cell consisting of hundreds of fibers, and the impact of the distribution of the contact points on the impedance change of an eddy current probe coil.

Index Terms—Carbon fiber reinforced polymer (CFRP), integral equation method, eddy current testing

I. INTRODUCTION

Carbon fiber reinforced polymers (CFRPs) are widely used in various fields, such as the aerospace and automotive industries. Since material defects can highly impact their mechanical properties and expected lifetime, the inspection and evaluation of material flaws is of key importance. Among many techniques (X-ray, ultrasound), eddy current testing (ECT) has been found to be effective in revealing fiber-related defects such as misorientation, fiber breaks, or misalignment [1][2].

Despite its wide application, the effect of the fiber distribution on the eddy current distribution and the yielded inhomogeneous 2D ECT pattern is still poorly understood [3]. In order to improve our understanding and enhance the detection methods, proper electromagnetic models should be established, which is quite challenging given the high aspect ratio of the configuration (cm long wires with a diameter of 7 μm). Though the latter can be treated in macroscopic finite element (FE) models using a homogenized anisotropic conductivity tensor [4], such models are inherently unable to take into account the local discrepancies in the fiber distribution. To overcome this problem, a recent paper proposed a modulation technique to emulate the fiber-rare and fiber-dense regions of the specimen [3]. A deeper analysis would require a microscopic fiber-level model of a smaller unit cell followed by some homogenization method as in [5].

A full FE analysis including inductive, capacitive, and resistive effects is highly resource demanding and not a feasible approach. We overcome this limitation by proposing an integral equation-based approach, where the carbon fibers are modeled as straight, cylindrical conductors, followed by a thin-wire approximation [6], that keeps the number of degrees of freedom (DoF) low. The occasional contact points between the fibers are treated as lumped resistors. In the Method of Moments (MoM) discretization, the effect of the latter can be effectively incorporated in the equation system by introducing some special basis functions, that enables the discontinuity of the

fibers' current distribution. With the suggested approach, we can analyze a unit cell made of a statistically representative number of fibers and investigate the effect of the fiber and contact distribution on the impedance change of a measuring coil.

II. INTEGRAL EQUATION FORMULATION

An $e^{j\omega t}$ time-harmonic excitation combined with the $A - \phi$ potential formalism and Lorenz' gauge decouples the Maxwell's equations into the free-space electric and magnetic inhomogeneous Helmholtz equations

$$\Delta \mathbf{A} + \omega^2 \mu_0 \varepsilon_0 \mathbf{A} = -\mu_0 \mathbf{J} \quad (1)$$

$$\Delta \phi + \omega^2 \mu_0 \varepsilon_0 \phi = -\frac{\rho}{\varepsilon_0}, \quad (2)$$

where $\mathbf{A}$ and ϕ denote the magnetic vectorpotential and electric scalar potential, $\mathbf{J}$ is the current density, ρ is the charge density, and ε_0 and μ_0 are the vacuum permittivity and permeability. The solutions of Eqs. (1) and (2) are given by the integrals

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_l \frac{I(\mathbf{r}') \mathbf{e}_l}{|\mathbf{r} - \mathbf{r}'|} e^{-j\frac{\omega}{c}|\mathbf{r} - \mathbf{r}'|} d\mathbf{l}' \quad (3)$$

$$\phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int_l \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} e^{-j\frac{\omega}{c}|\mathbf{r} - \mathbf{r}'|} d\mathbf{l}'. \quad (4)$$

In Eqs. (3) and (4), the usual phase shift is approximated by 1 due to the wavelength being much larger than the typical size of an ECT configuration, furthermore, $\mathbf{J}$ and ρ are suppressed into the fibers' axis l as a line current I and line charge density q , known as the *thin wire* approximation [7]. By prescribing the charge conservation and Ohm's law as

$$j\omega q(\mathbf{r}) + \frac{dI(\mathbf{r})}{d\mathbf{l}} = 0 \quad (5)$$

$$\mathbf{e}_l \cdot (-\nabla \phi - j\omega \mathbf{A})|_{\mathcal{F}} = \mathbf{e}_l \cdot \mathbf{E} = z(\omega) I(\mathbf{r}), \quad (6)$$

we obtain the integro-differential equation system (3)-(6) [6]. In (6), $\mathcal{F}$ denotes the surface of the fibers and $z(\omega)$ is the per-unit-length impedance of cylindrical conductors, obtained from Bessel functions, that incorporates the skin-effect as well.

III. NUMERICAL MODELING

After a 1-dimensional line segmentation is applied to the fibers, the charges and currents are expanded using piece-wise constant and piece-wise linear (triangular) basis functions, respectively. Using a Method of Moments (MoM) discretization with Dirac-delta test functions [7], the equation system can be re-written as $\mathbf{A} = \mathbf{M}\mathbf{I}$, $\mathbf{\Phi} = \mathbf{P}\mathbf{Q}$, $\mathbf{D}\mathbf{I} + \mathbf{j}\omega\mathbf{Q} = 0$, and $\mathbf{G}\mathbf{\Phi} + \mathbf{Z}\mathbf{I} + \mathbf{j}\omega\mathbf{A} = 0$.

To handle the discontinuity of the fibers' current distribution yielded by the presence of a contact point (modeled as a lumped resistor), we introduce a new basis function $\Lambda^i(x)$ as

$$\Lambda^i(x) = \begin{cases} \frac{1}{2} \frac{x-x_{i-1}}{x_i-x_{i-1}} & \text{if } x_{i-1} \leq x < x_i \\ -\frac{1}{2} \frac{x_{i+1}-x}{x_{i+1}-x_i} & \text{if } x_i < x \leq x_{i+1} \\ 0 & \text{otherwise,} \end{cases} \quad (7)$$

that has a jump at x_i . Hence, if a lumped resistor R_k with an unknown current $I_R^{(k)}$ is present between two fibers, connecting nodes x_i and x_j , using the basis functions $\Lambda^i(x)$ and $\Lambda^j(x)$ with coefficients $I_R^{(k)}$ and $-I_R^{(k)}$ enables the required jump in the current distribution. By taking into account the effects of $\Lambda^i(x)$ at the local element equations and prescribing Ohm's law as $\frac{\Phi(x_i) - \Phi(x_j)}{R_k} = I_R^{(k)}$ for all lumped resistors, the original equation system can be efficiently supplemented, that requires only $I_R^{(k)}$ as new unknowns.

IV. EXAMPLE

A 1-turn copper loop with $R = 1.25$ mm radius and $a = 0.05$ mm wire radius is placed at 0.4 mm lift-off over a single layer of 14 fibers ($\sigma = 59\,000$ S/m conductivity, 10 mm length, 0.1 mm radius and 0.25 mm distance) having quasirandom connection points. At the termination of the fibers, an $I = 0$ boundary condition is prescribed. The impedance change $\Delta Z = \Delta R + \mathbf{j}\Delta X$ of the loop in the 1 MHz ... 20 MHz frequency range is calculated and plotted in Fig. 1. A good agreement can be observed between the MoM and FEM results (obtained by COMSOL Multiphysics 6.2) as the increase of the real part and decrease of the imaginary part are clearly observable, however, the FEM model required 8.7×10^5 DoFs, whereas the MoM approach only needed 1638 DoFs.

V. CONCLUSION

A numerically efficient integral equation method has been proposed for the modeling of multiple thin fibers with randomly distributed connections in an ECT setup. The discrepancies between MoM and FEM will be further analyzed. In the final paper, the model domain will be rescaled to that of the carbon fibers (μm level), and a unit cell consisting of hundreds of fibers will be considered, focusing on the effect of the contact resistance and the spatial distribution of the contacts on the impedance change.

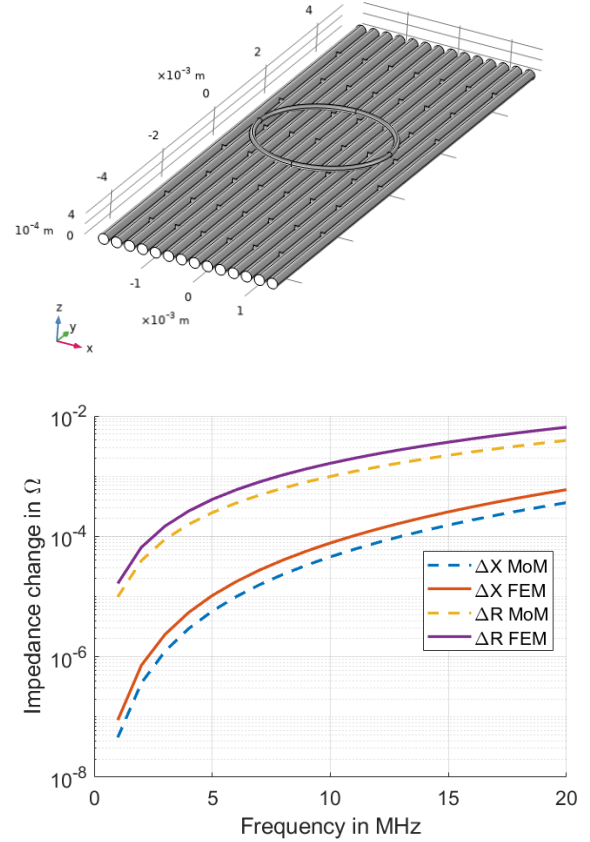


Fig. 1. Test example for a random connection of multiple fibers. Each pair of adjacent fibers has 4 contact points with 0.1 Ω contact resistance at a distance of 2.22 mm from each other. In the frequency curves, ΔR and $-\Delta X$ are plotted.

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