

3-D Magnetically Implicit Single-Field FDTD Scheme with Surface Impedance Absorbing Boundary Condition

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The magnetically implicit single-field finite-difference time-domain (MI-SF-FDTD) scheme was recently proposed as a promising direction of algorithm development to relax the stability condition of the explicit leapfrog FDTD scheme. However, its applicability is still limited to two-dimensional problems. In this work, the MI-SF-FDTD is extended to three-dimensional (3-D) open-region problems. For this purpose, the surface impedance absorbing boundary condition is incorporated in the 3-D MI-SF-FDTD scheme. Its numerical stability, computational efficiency and absorption performance are assessed in applications to closed and open problems.

Index Terms—Computational electromagnetics, finite difference methods, numerical stability, surface impedance.

I. INTRODUCTION

THE COURANT-FRIEDRICH-LEVY (CFL) condition of the finite-difference time-domain (FDTD) method [1] can severely limit the maximum stable time step size for the simulation of complex electromagnetic problems with very fine structures compared to wavelengths of interest. To remove this limitation, fully implicit FDTD methods with unconditional stability [2],[3] have been proposed and applied to 3-D problems. However, such methods often require solving a large system of linear equations at each time step or may suffer from accuracy degradation due to the splitting of the curl operator.

To model thin 3-D structures only in one direction, the 3-D hybrid implicit-explicit single-field FDTD method [4] based on the Crank-Nicolson method has been proposed. It has implicit time updating only for electric field. In contrast, the magnetically implicit single-field (MI-SF) FDTD method [5] based on the Newmark-beta method has been proposed. This scheme has implicit updating only for magnetic field. However, its applicability is limited to two-dimensional problems.

In this work, the MI-SF-FDTD scheme [5] with a relaxed stability condition is extended to 3-D open-region problems.

II. METHOD

Here we formulate a numerical scheme of the proposed method in the matrix-vector notation of the finite integration technique (FIT) [6], which is a consistent discretization method for numerically solving Maxwell's equations in integral form on the primary grid (G) and dual grid (G'). In the FIT, electric and magnetic grid voltages ($\mathbf{e}$, $\mathbf{h}$) defined on the grid system (G , G') are used as unknowns. For our formulation, we start with the following explicit leapfrog scheme (computationally equivalent to the classical Yee's algorithm): [6]

$$\mathbf{e}^{n+1/2} = \mathbf{e}^{n-1/2} + \Delta t \mathbf{D}_\epsilon^{-1} (\mathbf{C}^T \mathbf{h}^n - \mathbf{j}^n), \quad (1)$$

$$\mathbf{h}^{n+1} = \mathbf{h}^n - \Delta t \mathbf{D}_\mu^{-1} \mathbf{C} \mathbf{e}^{n+1/2}, \quad (2)$$

where Δt is the time step size, n is the time step index. $\mathbf{C}$ is the discrete curl matrix, $\mathbf{C}^T$ means the transpose of $\mathbf{C}$, $\mathbf{D}_\epsilon$ and $\mathbf{D}_\mu$ are the discrete permittivity and permeability matrices, their inverses have diagonal elements only.

Next, we define a temporal averaging of $\mathbf{h}$ with the so-called Newmark parameter β [7] as

$$\mathbf{h}_\beta^n = \beta \mathbf{h}^{n+1} + (1 - 2\beta) \mathbf{h}^n + \beta \mathbf{h}^{n-1} \quad (3)$$

and split the curl matrix

$$\mathbf{e}^{n+1/2} = \mathbf{e}^{n-1/2} + \Delta t \mathbf{D}_\epsilon^{-1} (\mathbf{C}_1^T \mathbf{h}_\beta^n + \mathbf{C}_2^T \mathbf{h}^n - \mathbf{j}^n) \quad (4)$$

as

$$\mathbf{C}^T = \mathbf{C}_1^T + \mathbf{C}_2^T, \quad (5)$$

where the subscript “1” means containing only the discrete partial differential operator for one direction of three coordinate axes (e.g., z -direction). Then, we change (2) at the previous time step ($n-1$) as

$$\mathbf{h}^n = \mathbf{h}^{n-1} - \Delta t \mathbf{D}_\mu^{-1} \mathbf{C} \mathbf{e}^{n-1/2}. \quad (6)$$

By taking the difference of (2) and (6), we have

$$\mathbf{h}^{n+1} - \mathbf{h}^n = \mathbf{h}^n - \mathbf{h}^{n-1} - \Delta t \mathbf{D}_\mu^{-1} \mathbf{C} (\mathbf{e}^{n+1/2} - \mathbf{e}^{n-1/2}). \quad (7)$$

When substituting (4) into (7) we can finally obtain the following three-level scheme:

$$\mathbf{A}_\beta \frac{\mathbf{h}^{n+1} - 2\mathbf{h}^n + \mathbf{h}^{n-1}}{\Delta t^2} = -\mathbf{D}_\mu^{-1} \mathbf{C} \mathbf{D}_\epsilon^{-1} (\mathbf{C}^T \mathbf{h}^n - \mathbf{j}^n), \quad (8)$$

$$\mathbf{A}_\beta = \mathbf{I} + \beta \Delta t^2 \mathbf{D}_\mu^{-1} \mathbf{C} \mathbf{D}_\epsilon^{-1} \mathbf{C}_1^T. \quad (9)$$

This scheme has two directionally implicit and one explicit updating equations, and requires only solving tridiagonal systems of linear equations.

When using a uniform cell size (L_x, L_y, L_z) on Cartesian grids and applying (3) into the z -direction, for $\beta \geq 1/4$, the numerical stability condition of (8) will be given as

$$c\Delta t \leq (1/L_x^2 + 1/L_y^2)^{-1/2}. \quad (10)$$

Note that L_z is not included in (10). Therefore, when L_z is much smaller than (L_x, L_y), (8) can be faster than an explicit update equation with $\beta=0$ in (9), because such an explicit scheme has the following stability condition:

$$c\Delta t \leq (1/L_x^2 + 1/L_y^2 + 1/L_z^2)^{-1/2}. \quad (11)$$

To model open-region problems, we apply the surface impedance absorbing boundary condition (SIABC) [8],[9] into

(8) in the current study. The subcell/conformal FDTD concept [10]-[12], which is not clearly mentioned in [8], is applied to only the nearest grid areas to the impedance boundary walls. In the FIT, this requires only the minor modification of (1) as

$$\mathbf{h}^{n+1} = \mathbf{h}^n - \Delta t \mathbf{D}_\mu^{-1} \left(\mathbf{C}_e \mathbf{h}^{n+\frac{1}{2}} \Big|_{\text{PEC}} + \mathbf{M}_Z \frac{\mathbf{h}^{n+1} + \mathbf{h}^n}{2} \right), \quad (12)$$

where $\mathbf{M}_Z$ is a material matrix related to the SIABC, and it consists of the wave impedance and grid lengths on cells partially filled with a PEC wall. Note that (12) can be regarded as a simplification of the update equation originally presented in [10]. In the case of the explicit leapfrog scheme (1), (12) with the SIABC, we can also find that (12) can lead to an explicit update equation including magnetic losses.

As the final step, by replacing (2) by (12) and by repeating the formulation strategy similar to (8), we can finally obtain the proposed scheme with the SIABC as follows:

$$\mathbf{A}_Z \mathbf{u}^{n+1} = \mathbf{B}_\beta \mathbf{h}^n + \Delta t \mathbf{D}_\mu^{-1} \mathbf{M}_Z \mathbf{h}^{n-1} + \Delta t^2 \mathbf{D}_\mu^{-1} \mathbf{C}_D \mathbf{e}^{-1} \mathbf{j}^n \quad (13)$$

$$\mathbf{h}^{n+1} = \mathbf{u}^{n+1} - \mathbf{h}^{n-1} \quad (14)$$

with

$$\mathbf{A}_Z = \mathbf{I} + \beta \Delta t^2 \mathbf{D}_\mu^{-1} \mathbf{C}_D \mathbf{e}^{-1} \mathbf{C}_1^T + \frac{1}{2} \Delta t \mathbf{D}_\mu^{-1} \mathbf{M}_Z \quad (15)$$

$$\mathbf{B}_\beta = 2\mathbf{I} - (1 - 2\beta) \Delta t^2 \mathbf{D}_\mu^{-1} \mathbf{C}_D \mathbf{e}^{-1} \mathbf{C}_1^T - \Delta t^2 \mathbf{D}_\mu^{-1} \mathbf{C}_D \mathbf{e}^{-1} \mathbf{C}_2^T \quad (16)$$

Note that by carefully choosing the sizes of nearest cells to the impedance boundary walls, time step reducing can be avoided in this scheme (8),(9).

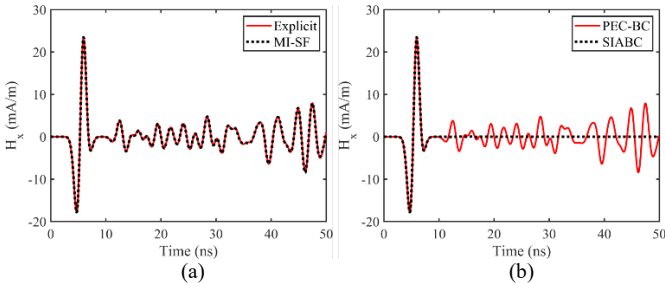


Fig. 1. Time waveform of H_x observed near a current source located at the center of the computational domain. (a) Explicit SF-FDTD and MI-SF-FDTD for PEC-BC, (b) PEC-BC and SIABC in MI-SF-FDTD.

III. NUMERICAL RESULTS

Here we verify the proposed 3-D MI-SF-FDTD scheme in its application to closed and open-region problems.

The first example is a 3-D closed cubic cavity (edge length $a=2.4\text{m}$) surrounded by the PEC walls. The fields are excited by a short current source (with a differential Gaussian waveform) located along z -axis at the center of the computational domain ($a \times a \times a$), which is discretized with $(L_x, L_y, L_z) = (0.03\text{m}, 0.03\text{m}, 0.01\text{m})$. Fig.1(a) illustrates the time waveforms of H_x at an observation point near the source simulated by the explicit SF-FDTD ($\beta=0$) and MI-SF-FDTD ($\beta=1/4$). The field oscillation, which originates from cavity resonances due to the existence of the PEC walls, is observed at a late time. Note that the upper limits of (11) and (10) are

used for the explicit SF-FDTD and MI-SF-FDTD, respectively. Good agreement between both results is obtained. The simulation takes 212s for the explicit SF-FDTD and 164s for MI-SF-FDTD. We can see that MI-SF-FDTD is stable even with the larger time step than the upper limit of (11) and faster than the explicit SF-FDTD by a factor of about 1.3.

The second example is an electromagnetic radiation from a current source in free space as an open-region problem. The dimensions and mesh sizes of a computational domain are the same as those of the first example. The SIABC is imposed on the terminated boundaries of the domain. Fig.1(b) demonstrates a difference between the waveforms of H_x for the two different BCs in the MI-SF-FDTD. No oscillation is observed in a late time after 10ns. This exemplifies the ability of the SIABC to absorb radiated waves in the context of the MI-SF-FDTD.

IV. CONCLUSION

The 3-D MI-SF-FDTD method has been formulated in the FIT matrix-vector notation. This scheme has two directionally implicit and one explicit time updating equations of magnetic field only. Its stability condition is more relaxed than that of a fully explicit variant. To model open-region problems, the FIT formulation of the SIABC using the subcell/conformal FDTD concept has been described and its extension to the 3-D MI-SF-FDTD has been also presented. Our scheme has been applied to the closed and open problems. It has been shown that the proposed 3-D scheme is stable under the upper limit of the relaxed stability condition and faster than its explicit variant.

REFERENCES

- [1] A. Taflov and S. C. Hagness, *Computational Electrodynamics: The Finite-Difference Time-Domain method*, 3rd ed. Norwood, MA, USA: Artech House, 2005.
- [2] G. Sun and C. W. Trueman, "An unconditionally-stable FDTD method based on the Crank-Nicolson scheme for solving the three-dimensional Maxwell's equations," *Electron. Lett.*, vol. 40, no. 10, pp. 589-590, May 2004.
- [3] T. Namiki, "3-D ADI-FDTD method-unconditionally stable time-domain algorithm for solving full vector Maxwell's equation," *IEEE Trans. Microw. Theory Techn.*, vol. 48, no. 10, pp. 1743-1748, Oct. 2000.
- [4] S. -M. Sadrpour, V. Nayyeri, M. Moradi, M. Soleimani and O. M. Ramahi, "A 3-D hybrid implicit explicit single-field finite difference time-domain method," *IEEE Trans. Electromagn. Compat.*, vol.61, no.3, pp.945-953, 2018.
- [5] K. Fujita, "Magnetically implicit single-field FDTD scheme based on the Newmark-Beta method," *IEEE Trans. Magn.*, vol. 60, no.3 article no. 7000205, March 2024.
- [6] T. Weiland, "Time domain electromagnetic field computation with finite difference method," *Int. J. Numer. Model.*, vol.9, pp.295-319, 1996.
- [7] N. M. Newmark, "A method of computation for structural dynamics," *J. Eng. Mech. Division*, vol. 85, no. 3, pp. 67-94, Jul. 1959.
- [8] Y. Mao et al., "Surface impedance absorbing boundary for terminating FDTD simulations," *ACES J.*, vol. 29, no. 12, pp. 1035-1046, 2014.
- [9] J. P. Berenger, "Theoretical investigation of the reflection from impedance absorbing boundary conditions," *IEEE Microw. Wireless Comp. Lett.*, vol. 28, no.7, pp. 543-545, 2018.
- [10] R. M. Mäkinen, H. D. Gersem, T. Weiland and M. A. Kivikoski, "Modeling of lossy curved surfaces in 3-D FIT/FDTD techniques," *IEEE Trans. Antennas Propag.*, vol. 54, no. 11, pp. 3490-3498, Nov. 2006.
- [11] K. Fujita, Y. Kochibe, and T. Namiki, "Numerical investigation of conformal ADI-FDTD schemes with second-order convergence," *IEICE Trans. Electron.*, vol. E93-C, no. 1, pp. 52-29, 2010.
- [12] K. Fujita, "Hybrid Newmark-conformal FDTD modeling of thin spoof plasmonic metamaterials," *J. Comput. Phys.*, vol.376, pp.390-410, 2019.