

Proper Orthogonal Decomposition for Model Order Reduction of Parametric Problems with Varying Geometry Based on the Interface Adaptive Mesh

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The proper orthogonal decomposition (POD) is commonly applied for the model order reduction (MOR) of parametric problems. The method generates a reduced basis from the left singular vectors of a matrix consisted by a set of snapshot solutions. The basis represents a lower dimensional subspace where the solutions lie on. Thus, it can be utilized for order reduction of the original problem. The singular value decomposition (SVD) of the snapshot matrix is a key step to obtain the reduced basis. However, a prerequisite of SVD is that the snapshots have the same degrees of freedom, so that they can consist a rectangular matrix. This prerequisite brings difficulty for the MOR based on POD for parametric problems involving varying geometric parameters. In this work, we propose to use an interface adaptive background mesh to assure the consistence of snapshot solution vector lengths with varying geometry. The method is applied in a preliminary numerical example that solves a parametric electrostatic problem with varying geometry.

Index Terms—Model Order Reduction, Proper Orthogonal Decomposition, Geometric Parameters, Interface Adaptive Mesh

I. INTRODUCTION

THE PARAMETRIC PROBLEMS with varying geometric parameters needs to be solved for the optimization design or behavior modeling of equipment or devices. In the process, the numerically discretized physics equations, such as Maxwell's equations for electromagnetics analysis, are solved repeatedly for a variety of samples selected from the design space. Projection-based model order reduction (MOR) methods, such as proper orthogonal decomposition (POD) [1,2], are commonly used to reduce the computational cost for solving the parametric problems.

POD utilizes the left singular vectors of a snapshot matrix consisting by pre-calculated solution vectors from the traditional numerical methods (such as FEM, FIT, etc.) as the reduced basis, and then the original discrete equations can be projected in a sub-space with lower dimensionality, so that the scale of the algebraic equations can be reduced. However, it is difficult to apply POD to parametric problems with varying geometry. Since when the geometric model is changed, the mesh and the degrees of freedom to be solved are usually changed subsequently. This leads to inconsistent snapshot resolution vector lengths, and thus the rectangular snapshot matrix cannot be naturally formed. In this work, we propose to use an interface mesh adaption technique to tackle the difficulty for the application of POD in the parametric problems with varying geometry. The method was firstly brought up to solve movement involved eddy current problems [3].

In this digest, we applied the proposed approach for the MOR of a parametric electrostatic problem considering the parasitic capacitance extraction of interconnects with varying positions and widths. The simulation results from the reduced model can be used to construct pattern libraries for the on-the-fly parasitic extraction in integrated circuits [4].

II. METHOD

A. MOR based on POD

Considering the discretized algebraic system for a parametric problem

$$K(\mathbf{p})\mathbf{x} = \mathbf{b}(\mathbf{p}), \quad (1)$$

where $\mathbf{p}$ denotes a parameter vector which consists of various parameters, $K(\mathbf{p})$ and $\mathbf{b}(\mathbf{p})$ denote the system matrix and the right-hand side (RHS) vector corresponding with $\mathbf{p}$ respectively. Giving a sampling of the parametric space $[\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_{n_s}]$, a snapshot matrix $S = [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{n_s}]$ of dimension $N \times n_s$ can be obtained by solving (1), where n_s is the number of samples usually small and N is the number of DoFs. Matrix Φ , whose columns are the dominant left singular vectors of S , can be utilized as a reduced basis for the construction of the following reduced problem

$$\Phi^T K(\mathbf{p}) \mathbf{x}_r = \Phi^T \mathbf{b}(\mathbf{p}), \quad (2)$$

where $\mathbf{x}_r = \Phi \mathbf{x}$. We denote the number of DoFs of (2) as n_r , which is equal to or less than n_s , much smaller than the original problem scale N . It is notable that the vector of snapshot solutions needs to be with the same length to form the snapshot matrix, and this condition is hard to be fulfilled when $\mathbf{p}$ contains geometric parameters.

B. Interface Mesh Adaption

In this work, we propose to use an interface mesh adaption technique to assure the same number of DoF when the geometry varies, so that the length of the snapshot solution vectors is consistent and POD can be utilized for MOR. Firstly, an initial background mesh is generated in the whole computational domain. The varying geometry can be considered as an upper layer of the background. For any given set of parameters, the background mesh is adapted to the interface between the varying geometry parts and the background without changing

the topology of the background mesh [3], and thus the DoF system can keep unchanged and MOR based on POD can be implemented. Fig.1 shows an example of interface mesh adaption.

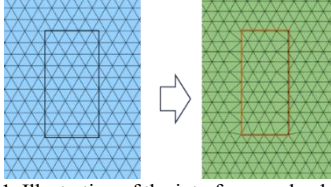


Fig. 1. Illustration of the interface mesh adaption.

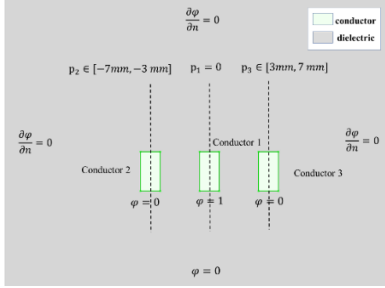


Fig. 2. Illustration of the numerical example.

III. NUMERICAL EXAMPLE

We consider the parametric electrostatic problem as illustrated in Fig. 2. Three conductors are placed in a computational domain. Conductor 1 is considered as the main conductor with an electric potential 1V and placed at position 0. The electric potential on Conductor 2 and 3 as well as the ground is set to 0V. The size of all three conductors is 2mm×4mm. The position of conductors 2 and conductor 3 are denoted as p_2 and p_3 and considered as varying parameters changing in ranges $[-7\text{mm}, -3\text{mm}]$ and $[3\text{mm}, 7\text{mm}]$ respectively.

The finite element method (FEM) is used for solving the electrostatic problem

$$\nabla \cdot \epsilon \nabla \phi = 0 \quad (3)$$

where ϕ is the electric potential and ϵ is the electric permittivity. Firstly, we solved for 25 snapshot solutions for combinations of the two parameters in the set $\{(p_2, p_3): p_2 \in \{-3, -4, -5, -6, -7\}, p_3 \in \{3, 4, 5, 6, 7\}\}$. The decaying of the singular values of the snapshot matrix is presented in Fig. 3(a).

According to a truncation criterion $\frac{\sum_{i=1}^{n_r} s_i}{\sum_{i=1}^{n_s} s_i} < 1 - e^{-4}$, where s_i denotes the i th singular values of the snapshot matrix, 11 basis vectors corresponding to the 11 smallest singular values are omitted. Subsequently, a reduced basis with 14 dimensions is obtained. Then, we solve for the reduced problem at $p_2 = -4.5$ and $p_3 = 5.5$. Fig. 4 shows the electric potential field distribution recovered from the solution of the reduced problem and the reduced basis. The self-capacitance and coupling-capacitance derived from the FEM and POD solutions are compared in Table. 1. Results show that the relative error of the self-capacitance is 0.19% between POD and FEM, while the relative error of coupling-capacitance is around 1% between POD and FEM. The computational time of POD is 0.006 (with 14 DoFs) while the cost for FEM is 0.026s (with 3841 DoFs) on the same workstation. Thus, the computational cost can be

evidently saved when we need to solve for a large number of samples for building the capacitance pattern models.

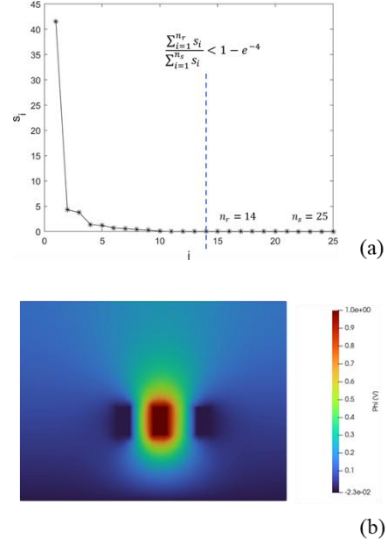


Fig. 3. (a) The decaying of the singular values of the snapshots matrix; (b) The distribution of electric potential recovered from the reduced problem solution and POD basis at $p_2 = -4.5\text{mm}$ and $p_3 = 5.5\text{mm}$

TABLE I
COMPARISON OF SELF AND COUPLING CAPACITANCE
CALCULATED BY FEM AND POD

Quantities	FEM	POD	RELATIVE ERROR
$C_{11} [fF]$	3.995e+04	4.0025e+04	0.19%
$C_{12} [fF]$	-1.8615e+04	-1.8378e+04	1.27%
$C_{13} [fF]$	-1.4998e+04	-1.4885e+04	0.75%

IV. CONCLUSION

In this work, we propose to use an interface mesh adaption method for the generation of reduced basis with POD for parametric problems with varying geometry. We take a preliminary example with a two-dimensional parametric space for the validation of the proposed method. In the full paper, numerical examples with higher dimension parametric space and a generalization of the proposed approach to 3D will be addressed.

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