

Enhanced Low-Frequency Dosimetry Simulations with the Energy-Variation Maximum Entropy Snapshot Sampling

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Projection-based model order reduction enables efficient high-fidelity low-frequency dosimetry simulations based on pre-computed solution vectors of the large system. Since most of these vectors provide proximal information about the system, the information-theoretic framework called maximum entropy snapshot sampling can be used to reduce them to the only essential ones from which a reduced basis can be constructed. Although this framework is memory efficient, it has a computational bottleneck in constructing the required recurrence matrix due to the use of state-recurrence analysis. Therefore, this work aims to improve this framework by using energy-variation analysis.

Index Terms—energy variation, low-frequency dosimetry, model order reduction, maximum entropy snapshot sampling

I. INTRODUCTION

LOW-FREQUENCY dosimetry simulations are essential to ensure protection from low-frequency magnetic fields emitted by inductive charging systems used in public spaces for wireless charging of battery-powered electric transport vehicles, as shown in Fig. 1. In this context, the electric field induced in the exposed human body should not exceed the exposure limits established by the International Commission on Non-Ionizing Radiation Protection (ICNIRP) [1]. For comprehensive exposure assessments, the scalar-potential finite difference (SPFD) scheme [2] can be used, in which the *Poisson* system of linear algebraic equations is solved

$$\nabla \cdot [\sigma \nabla \psi] = \nabla \cdot [-j\omega \sigma \vec{A}], \quad (1)$$

with the electric scalar potential ψ and the electric conductivity of human body tissues σ . With an ICNIRP-recommended voxel resolution of at least 2 mm, (1) typically has a size of millions to tens of millions of degrees of freedom (DOF), so such low-frequency dosimetry simulations are computationally intensive.

Projection-based model order reduction techniques, such as reduced basis methods (RBMs) like the proper orthogonal decomposition (POD) [3], allow the generation of reduced order models (ROMs) of (1) based on *a posteriori* high-fidelity solution vectors $\psi_\nu \in \mathbb{C}^m$ parameterized by the position of the human model. In this way, high-fidelity solution vectors ψ of (1) can be approximated in near real-time and with efficient memory usage, even on mobile devices [4]. Within the framework of RBMs, the singular value decomposition (SVD) of the so-called snapshot matrix

$$\Psi = (\psi_1 | \psi_2 | \dots | \psi_n) \in \mathbb{C}^{m \times n}, \quad (2)$$

which contains column-wise n high-fidelity solution vectors ψ_ν of (1), is a commonly applied approach to construct a reduced basis of Ψ . According the SVD, Ψ is factorized as

$$\Psi = \mathbf{U} \Sigma \mathbf{V}^* \quad (3)$$

with the unitary matrices $\mathbf{U} \in \mathbb{C}^{m \times m}$ and $\mathbf{V} \in \mathbb{C}^{n \times n}$ containing the left and right singular vectors of Ψ , respectively, together with the diagonal matrix $\Sigma \in \mathbb{R}^{m \times n}$ whose main diagonal contains the singular values $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_n \geq 0$.

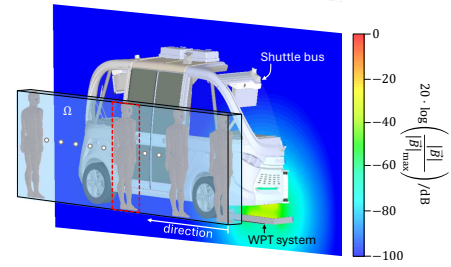


Fig. 1. Low-frequency magnetic field simulation with a MILAS shuttle bus [5], in which snapshots are taken in domain Ω at 50 mm intervals along a predefined direction.

Factorizing high-dimensional snapshot matrices Ψ using SVD usually involves significant computational and memory overhead to construct a reduced basis from the first $\ell \ll m$ column vectors of $\mathbf{U}$. Even a computationally faster approach that approximates the SVD based on a random sample of $k \geq \ell$ column vectors of Ψ , called randomized SVD (rSVD) [6], features a relatively high memory footprint. It has been shown in [7] that a reduced basis of Ψ can be efficiently constructed with the maximum entropy snapshot sampling (MESS) framework [8] with minimal memory footprint by first reducing the number of ψ_ν in Ψ to those with essential physical and topological information regarding (1). The MESS presented therein is based on state-recurrence analysis, which leads to computational complexity in $\mathcal{O}(mn^2)$ in the construction of the required recurrence matrix $\mathbf{R}_\epsilon \in \{0,1\}^{n \times n}$ and thus is a bottleneck of this framework. This work proposes an enhanced MESS framework based on energy-variation analysis [9] (EV-MESS) to reduce the computational complexity.

II. MAXIMUM ENTROPY SNAPSHOT SAMPLING

The original MESS framework is based on state-recurrence analysis to identify information-relevant snapshots ψ in Ψ based on $\mathbf{R}_\epsilon$, a binary representation of a ϵ -threshold normalized distance matrix. More precisely, $\mathbf{R}_\epsilon$ is defined by

$$\mathbf{R}_\epsilon(i, j) = \Theta(\epsilon - d(\psi_i, \psi_j)) \quad \forall i, j \in [n], \quad (4)$$

where $\Theta : \mathbb{R} \rightarrow \{0,1\}$ is the Heaviside step function and $d(\psi_i, \psi_j) = \|\psi_i - \psi_j\|_2$. From a visual point of view, $\epsilon > 0$ defines the radius of clustering orbits with snapshots ψ_ν as centroids $B_\epsilon(\psi_\nu) = \{\mathbf{y} \in \mathbb{R}^m \mid \|\psi_\nu - \mathbf{y}\|_2 < \epsilon\}$, clustering

the snapshots into proximal information snapshots ($\varepsilon \rightarrow 0$), which are in $B_\varepsilon(\psi_\nu)$, and non-proximal ones. However, the construction of $\mathbf{R}_\varepsilon$ in terms of state-recurrence analysis is a bottleneck in the original MESS framework due to its computational complexity of $\mathcal{O}(mn^2)$.

As an alternative to state-recurrences, energy-recurrence states can be considered in the context of energy-variation analysis to generate a recurrence matrix $\mathbf{W}_\varepsilon \in \{0, 1\}^{(n-1) \times (n-1)}$. To construct $\mathbf{W}_\varepsilon$, the energy vector $\mathbf{e} \in \mathbb{R}^{n-1}$ has to be determined whose w -th component is $e_w = [d(\psi_{w+1}, \psi_w)]^2$, with $w \in [n-1]$. Along with $\varepsilon > 0$, $\mathbf{W}_\varepsilon$ is defined as

$$\mathbf{W}_\varepsilon(i, j) = \Theta(\varepsilon - |e_i - e_j|) \quad \forall i, j \in [n-1] \quad (5)$$

with the Heaviside step function $\Theta: \mathbb{R} \rightarrow \{0, 1\}$. Thus, constructing the energy-recurrence matrix $\mathbf{W}_\varepsilon$ has a computational complexity in $\mathcal{O}(\max\{n^2, mn\})$ lower than that of $\mathbf{R}_\varepsilon$.

Based on $\mathbf{W}_\varepsilon$, the indices $\nu \in [n]$ of whose snapshots ψ_ν in Ψ can be determined using the colon notation, which increase the so-called ε -Frobenius entropy

$$\eta_\nu = \eta_\varepsilon(\Psi(:, 1:\nu)) = -\log\left(\frac{1}{\nu^2} \|\mathbf{W}_\varepsilon(1:\nu, 1:\nu)\|_F^2\right) \quad (6)$$

with the Frobenius norm $\|\cdot\|_F$ and the ε -Frobenius potential

$$v_\nu = v_\varepsilon(\Psi(:, 1:\nu)) = \frac{1}{\nu^2} \|\mathbf{W}_\varepsilon(1:\nu, 1:\nu)\|_F^2, \quad (7)$$

where v_ε is the so-called information potential. By its definition, v_ν is governed by the initial-value problem

$$v_1 = 1, \quad v_{\nu+1} = \frac{\nu^2}{(\nu+1)^2} v_\nu + \frac{1}{(\nu+1)^2} \delta_\nu, \quad (8)$$

where δ_ν is defined considering the symmetry of $\mathbf{W}_\varepsilon$ as

$$\delta_\nu = 2 \sum_{k=1}^{\nu} \mathbf{W}_\varepsilon(\nu+1, k) + 1. \quad (9)$$

Therefore, the local (in time) entropy increment condition $v_{\nu+1} < v_\nu$ becomes

$$\delta_\nu < (2\nu+1)v_\nu. \quad (10)$$

By recursively evaluating (10), all snapshots $\psi_\nu \forall \nu \in [n-1]$ are identified, which increase the ε -Frobenius entropy. Finally, a reduced basis of Ψ is obtained by applying any orthonormalization scheme on the reduced collection of snapshots.

III. PRELIMINARY NUMERICAL RESULTS

To evaluate the MESS framework using energy-variation analysis, 100 snapshots ψ_ν of (1) in Ω , see Fig. 1, are used, which are taken according to a predefined orientation with a fixed direction at a uniform distance of 50 mm. Each ψ_ν represents the scalar potential for the human body model *Duke* [10] with a voxel edge resolution of 2 mm, resulting in (1) with approximately $m = 10^7$ DOF. The MESS-required ε -threshold parameter is set to $\varepsilon = 0.01$, resulting in a reduced subset Ψ_r of Ψ containing 29 essential snapshots ψ_ν , whose indices are highlighted in Fig. 2. By orthonormalizing Ψ_r , a reduced basis is obtained, which is used to form a ROM of (1) with only 29 DOF. In the context of low-frequency dosimetry simulations using SPFD, the MESS-based ROM provides an electric field distribution, as shown in Fig. 3, for the human

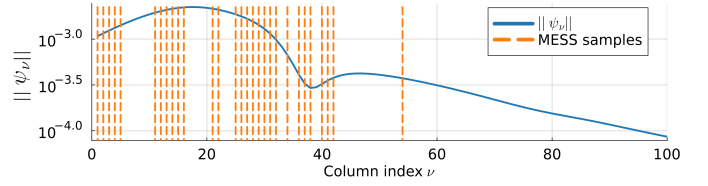


Fig. 2. $\|\psi_\nu\|$ versus the column index ν and the MESS samples using state-recurrence and energy-variation analysis.

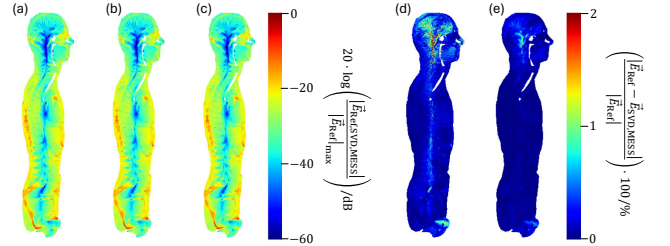


Fig. 3. In (a), reference electric field for the human voxel model highlighted in Fig. 1. The SVD and MESS-based ROM solutions are shown in (b) and (c), respectively, with relative differences shown in (d) and (e), respectively.

TABLE I
BENCHMARKS FOR COMPUTING 29 BASIS VECTORS FROM Ψ USING SVD, rSVD, MESS, AND EV-MESS ON AN AMD RYZEN 7900X CPU

	SVD	rSVD	MESS ^{a,b}	EV-MESS ^b
Computation time [s]	147.06	17.07	66.96	14.99
Memory requirement [GB]	21.32	49.06	10.31	18.84

^a ε -threshold parameter set to $\varepsilon = 0.03$.

^b Singular value decomposition is used as orthonormalization scheme.

model highlighted in Fig. 1. This distribution is as accurate as those based on 29 basis vectors provided by the SVD of Ψ . The benchmark results in Table I show that the enhanced EV-MESS framework generates a reduced basis of Ψ faster than the original MESS framework, SVD, and rSVD in the used application. Regarding memory consumption, the EV-MESS framework outperforms SVD and rSVD but is inferior to the original MESS framework.

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