Model Order Reduction for Fast Core-Loss Calculations in Synchronous Machines

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This paper investigates the application of model order reduction techniques for efficient implementation of a core-loss model in synchronous machine simulations. The machine state-space model is constructed by solving multiple static finite element (FE) simulations, representing the winding flux linkages and flux-density distribution in the FE nodes as functions of the stator currents and rotor angle, and storing these results into lookup-tables. Three different approaches for implementing the core-loss model are explored. The proper orthogonal decomposition and discrete empirical interpolation method are employed to reduce the computational complexity of these implementations while maintaining model accuracy. The reduced-order model demonstrates good agreement with the full model, achieving an 18-fold increase in the execution speed and 88% reduction in the pre-computation time.

***Index Terms*—Core loss, lookup-tables, reduced order model, synchronous machine.**

I. Introduction

THE performance assessment of electric vehicles over extended driving cycles requires fast and accurate models for all the subsystems and their associated losses. Among powertrain components, the electric motor contributes the highest share of losses [1]. The calculation of the DC copper loss is directly related to the stator current. However, the core-loss calculation is more difficult.

The finite element (FE) models offer highly accurate loss calculations with the inclusion of the material non-linearities. However, these models are computationally intensive and unsuitable for long simulations. Another approach to reduce the computation burden of the FE models is to use the FE based state-space (SS) models. The machine is modeled by storing the solution of multiple static FE simulations into lookup-tables (LUTs). As these LUTs are derived from FE solutions, they can achieve accuracy comparable to FE model while keeping efficient computation time [2]. Moreover, the core loss can be implemented as a separate LUT depending on the model states for efficient evaluation [3].

The core loss model implemented in [3] is a general post-processing model applicable for any non-linear iron-loss model that expresses the instantaneous loss density as a function of the flux density and its time derivative. The authors introduced three different approaches to integrate that loss model to the motor SS model. The model implementations show fast execution time and good match with the experimental results. However, the time taken to construct that model can be challenging, especially when a large section of the machine with a high number of FE nodes is modeled. In this paper, different model order reduction (MOR) techniques are applied to the different approaches in [3] to efficiently implement the core-loss model.

II. Methods

A brief description of the motor SS model and the employed core-loss model is described here. The complete derivation can be found in [3].

A. State-Space Model Including Core Loss

Starting from the phase voltage equation

$$\boldsymbol{u}_{\mathrm{abc}} = R_{\mathrm{s}}\boldsymbol{i}_{\mathrm{abc}} + \frac{d\boldsymbol{\psi}_{\mathrm{abc}}\left(\boldsymbol{i}_{\mathrm{abc}}, \alpha\right)}{dt}, \tag{1}$$

and by expressing the flux linkage time derivative using the chain rule, the machine SS model can be derived as

$$\begin{aligned}\frac{d\boldsymbol{i}_{\mathrm{ab}}}{dt} = \left(\frac{\partial(\boldsymbol{K}\boldsymbol{\psi}_{\mathrm{abc}})}{\partial \boldsymbol{i}_{\mathrm{ab}}}\right)^{-1} \Bigg[\boldsymbol{Q}\boldsymbol{U}_{123} - R_{\mathrm{s}}\boldsymbol{K}\boldsymbol{K}^{\mathrm{T}}\boldsymbol{i}_{\mathrm{ab}} \\ - \boldsymbol{K}\frac{\partial(\boldsymbol{K}\boldsymbol{\psi}_{\mathrm{abc}})}{\partial\alpha}\omega\Bigg].\end{aligned} \tag{2}$$

The employed core-loss model expresses the power-loss density p as a function of the flux-density vector $\boldsymbol{B}$ and its time derivative as $p\left(\boldsymbol{B}, \partial\boldsymbol{B}/\partial t\right)$. The $\boldsymbol{B}$ distribution in all FE integration points i is stored in a separate LUT at each value of the state variables while running the static FE solver. The total core loss is obtained by integrating the core-loss density over the FE integration points of the core region Ω

$$P = \int_{\Omega} p\left(\boldsymbol{B}, \frac{\partial \boldsymbol{B}}{\partial t}\right) d\Omega = \sum_i w_i p\left(\boldsymbol{B}_i, \frac{\partial \boldsymbol{B}_i}{\partial t}\right), \tag{3}$$

where i sums over the FE integration points and w_i are the integration weights.

Three different approaches for implementing the core-loss model (3) in Simulink are discussed in [3]. The first approach represents the flux-density time derivative using the chain rule with respect to the model states, while the second approach implements it using the Simulink built-in time-derivative block. The third approach pre-calculates and stores the core loss in a separate LUT. Approaches 1 and 2 were comparatively slower than Approach 3 due to the need for interpolating $\boldsymbol{B}$ and $\partial\boldsymbol{B}/\partial t$ data at each time step. Furthermore, Approach 3 required time to pre-calculate the loss LUT. Nevertheless, the three approaches showed acceptable accuracy when compared to time-stepping FE simulation and the experimental measurement results.

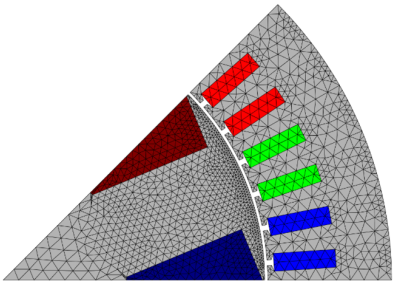


Fig. 1. FE mesh of 1/8 segment of the machine with 1476 elements and 871 nodes.

B. Model Order Reduction

To overcome previously mentioned performance issues, MOR techniques are employed to compress the $\boldsymbol{B}$ data, leading to significant reduction in its size. This leads to faster interpolation in Simulink and more efficient pre-calculation of the loss LUT.

Proper orthogonal decomposition (POD) decomposes the data into its spatial and temporal components by expressing it as a sum of spatial functions multiplied by time-dependent coefficients as

$$\boldsymbol{B}(x,t) = \sum_{j=1}^{\infty} \phi_j(x) a_j(t), \tag{4}$$

where $\phi_j(x)$ is called POD modes and $a_j(t)$ captures all the time-dependent behavior. The optimal POD modes are obtained from the singular value decomposition of the snapshot matrix, which is constructed by solving the full order model (FOM) multiple times at different operating conditions. The obtained POD modes have much lower dimensions than the FOM. However, when the non-linear part of the system is significant, POD calculations become expensive since the non-linear terms are computed with the FOM dimensions. Therefor, discrete empirical interpolation method (DEIM) is used alongside POD to handle the non-linear terms [4].

To eliminate the dependency of the reduced solution on the FOM dimension, DEIM suggests approximating the solution of the non-linear term instead of directly evaluating it [5]. This is achieved by selecting specific interpolation indices to define an interpolation-based projection in a nearly ℓ_2 optimal subspace, allowing the non-linearity to be approximated without the expense of orthogonal projection. More details about the method and implemented algorithm will be provided in the full paper.

III. Application and Results

The MOR techniques discussed earlier are applied to reduce the computation burden of the implemented core-loss approaches discussed in Section II-A. The reduced order model (ROM) results are verified against the results of the FOM and the 2-D time-stepping FE simulation results for a 200 kW synchronous motor excited by inverter supply. The FE mesh of the modeled section is shown in Fig. 1. More details are provided in [3].

Table I presents a comparison between the core loss obtained from different approaches using full and reduced $\boldsymbol{B}$ data with the time-stepping FE simulation at rated conditions. Comparing the simulation time per time step for Approaches 1 and 2 with both full and reduced $\boldsymbol{B}$ data shows clearly the benefit of employing the MOR techniques. The execution times for Approaches 1 and 2 are reduced by factors of 18 and 11, respectively, compared to the execution times when using the full $\boldsymbol{B}$ data, while maintaining the good match with the FOM results. On the other hand, the execution time for Approach 3 does not change as the same amount of data is interpolated in both cases. However, the pre-computation time for the loss LUT is reduced by almost 88% compared to the time required when the full data is used.

In the full paper, more explanations of the methods, detailed assessment of the performance and comparison with the experimental results will be presented.

TABLE I
Comparison between the core loss obtained from the full $\boldsymbol{B}$ data and reduced $\boldsymbol{B}$ data with the FE calculations at rated load

	Approach 1		Approach 2		Approach 3		FE
	FOM	ROM	FOM	ROM	FOM	ROM	
$\boldsymbol{B}$ data LUT construction time (min)	98		98		98		-
Loss LUT Pre-computing time in (3) (min)	-	-	-	-	125	15	-
Simulation time per time step (ms)	28	1.5	5	0.45	0.15	0.15	18
Core loss (W)	1438	1475	1515	1521	1533	1537	1502

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