

Finite-Element Based State-Space Modeling of AC Winding Losses in Synchronous Machines

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This paper proposes a finite element (FE) based state-space model for AC winding losses in synchronous machines to be implemented in Simulink. The model is constructed by storing the solutions of multiple static FE simulations into lookup-tables to express the magnetic vector potential on the boundaries of the slot region as a function of stator currents and rotor position. The geometry-dependent FE matrices derived from the mesh are computed only once, while the dynamic simulation evaluates AC winding losses at each time step based on the instantaneous operating conditions. The Simulink implementation of the proposed AC winding loss model matches well with the time-stepping FE simulation results, while achieving a 105-fold speedup in computation.

Index Terms—AC losses, lookup-tables, state-space model, synchronous machine, windings.

I. INTRODUCTION

AC winding losses are a significant source of power loss in electrical machines, particularly at high operating speeds. Accurate modeling of these losses is essential for evaluating machine efficiency over an extended operating range, such as during a full driving cycle simulation. Methods for calculating AC winding losses can be categorized into three approaches [1]: finite element (FE)-based models [2], analytical models [3], and hybrid models [4]. FE-based models provide highly accurate results. However, they are computationally intensive and impractical for long dynamic simulations. Analytical models offer fast calculation of the losses, but their accuracy is often compromised due to inherent assumptions and approximations. Hybrid models try to combine the strengths of both approaches by solving FE simulations at selected operating points and use the analytical models to generalize the results across a wider range of operating conditions.

A FE-based state-space (SS) model for synchronous machines, including core loss, was introduced in [5]. This model was constructed by solving multiple static FE simulations, representing the winding flux linkages and the flux-density distribution in the FE integration points as functions of stator currents and rotor angle, and storing these results into lookup-tables (LUTs). The core loss was pre-computed and stored into a separate LUT for fast computation in Simulink. The model demonstrated good accuracy and fast execution time. The same procedure will be followed to calculate the AC winding losses in the solid stator conductors of a synchronous machine. However, modeling such winding phenomena, which include internal states due to eddy currents, cannot be achieved using only the SS model based on static FE solutions. Therefore, additional states are introduced.

In this paper, the SS model is extended with a stator winding loss model that accounts for skin and proximity effects in series conductors. The winding loss model is a 2-D FE formulation of a single slot coupled with the circuit equations of the stator winding with solid conductors. The resulting differential-algebraic equation (DAE) system can be

implemented as a computationally efficient block in Simulink. The winding loss model is driven by the conductor currents and the values of the out-of-plane component of the vector potential on the boundary of the slot. A separate LUT is used to store these boundary values, which are obtained from the static FE solutions, as functions of stator currents and rotor position. Moreover, proper orthogonal decomposition (POD) technique is used to reduce the model complexity and speedup its computation speed.

II. METHODS

The FE system for the slot can be written as

$$\begin{bmatrix} T_{cc} & 0 & 0 \\ 0 & 0 & 0 \\ -F^T & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} a_c \\ a_n \\ e \end{bmatrix} = \begin{bmatrix} -S_{cc} & -S_{cn} & -F \\ -S_{nc} & -S_{nn} & 0 \\ 0 & 0 & l_z/R \end{bmatrix} \begin{bmatrix} a_c \\ a_n \\ e \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ P & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \hat{a} \\ i \end{bmatrix}, \quad (1)$$

where a_c and a_n contain the nodal values of the vector potential in the conductors and the non-conducting regions, respectively, e contains the out-of-plane components of the electric fields in the conductors, and $\hat{a}$ and i contain the Dirichlet conditions and the conductor current that are used as the inputs. S and T are the FE stiffness and damping matrices, F is a matrix for imposing the field sources and calculating the induced voltages in the conductors, P is a selection matrix for assigning the Dirichlet conditions on the boundary nodes, R contains the DC resistances of the conductor regions and l_z is the out-of-plane length. After the system has been solved, the eddy-current loss per unit length in the conductors can be calculated using da_c/dt and e as

$$p = \begin{bmatrix} \frac{da_c}{dt} \\ e \end{bmatrix}^T \begin{bmatrix} T_{cc} & F \\ -F^T & -l_z/R \end{bmatrix} \begin{bmatrix} \frac{da_c}{dt} \\ e \end{bmatrix}. \quad (2)$$

It turns out that the DAE system (1) has a high nilpotency index and is not solvable as such in Simulink. By eliminating

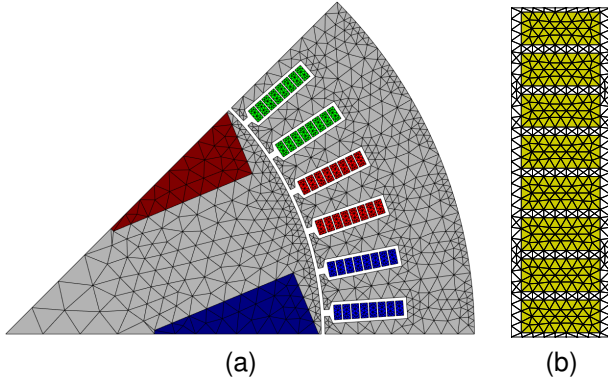


Fig. 1. Finite element mesh of the machine under study. (a) FE mesh of 1/8 segment of the machine. (b) FE mesh of the slot with 8 solid conductors.

e from the system and adding $\dot{a}_c = da_c/dt$ as an additional variable, we get

$$\begin{bmatrix} T_{cc} - FF^T & 0 & 0 \\ 0 & 0 & 0 \\ I & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} a_c \\ a_n \\ \dot{a}_c \end{bmatrix} = \begin{bmatrix} S_{cc} & S_{cn} & 0 \\ S_{nc} & S_{nn} & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} a_c \\ a_n \\ \dot{a}_c \end{bmatrix} + \begin{bmatrix} 0 & FR/l_z \\ P & 0 \end{bmatrix} \begin{bmatrix} \hat{a} \\ i \end{bmatrix}, \quad (3)$$

which becomes solvable. For loss calculation, da_c/dt and e can be solved from

$$\begin{bmatrix} \frac{da_c}{dt} \\ e \end{bmatrix} = \begin{bmatrix} 0 & 0 & I \\ 0 & 0 & -F^T \end{bmatrix} \begin{bmatrix} a_c \\ a_n \\ \dot{a}_c \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -R/l_z \end{bmatrix} \begin{bmatrix} \hat{a} \\ i \end{bmatrix}. \quad (4)$$

Once the solution has been obtained, systems (3) and (4) can be written as a descriptor state-space model

$$\begin{aligned} E \frac{dx}{dt} &= Ax + Bu \\ y &= Cx + Du, \end{aligned} \quad (5)$$

which can be straightforwardly implemented in Simulink.

III. APPLICATION AND RESULTS

The proposed model is verified against a 2-D time-stepping FE simulation using a 200 kW wound-field synchronous motor equipped with 8 solid conductors per slot. Fig. 1a shows the FE mesh of the modeled sector of the machine, while Fig. 1b shows a single slot mesh with 8 solid conductors. The machine has 48 slots, 4 pole pairs and its rated speed is 3000 rpm.

A separate model is constructed for each individual slot within the symmetry section in Fig. 1a, and the corresponding inputs -Dirichlet conditions and currents- are fed to each slot independently. The total loss is then obtained by summing up the loss for those six slots and multiply by the number of symmetry sections. For efficient implementation of that model, POD is used to reduce the model complexity and increase the execution speed.

Fig. 2 compares the total copper loss obtained from the Simulink model with that from the time-stepping FE model. Below the base speed (200 Hz), the ratio between the voltage and frequency is kept constant. Above the base speed,

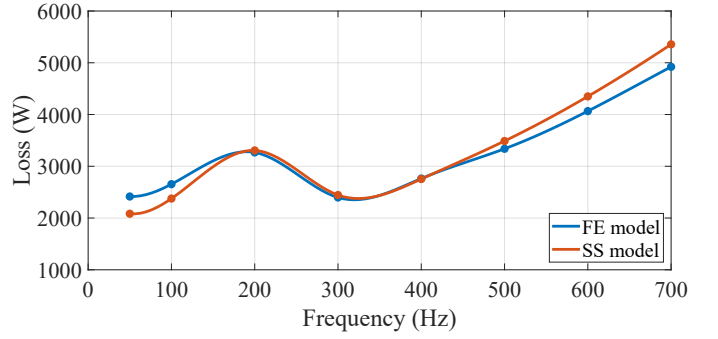


Fig. 2. Comparison between total copper loss computed from time-stepping FE simulation and Simulink implementation of the proposed model across different fundamental frequencies.

however, the voltage is kept constant and the frequency is increased. The figure demonstrates a good agreement between both models. From a computational efficiency perspective, the FE model requires 45 ms per time step, whereas the Simulink implementation requires only 0.42 ms per time step, which is approximately 105 times faster than the FE model.

In the full paper, a detailed explanation of the methods, an extended comparison with the time-stepping FE simulation and a comprehensive assessment of the performance and accuracy of the proposed modeling approach will be presented.

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