

Lumped Element Implementation in an EFIE Technique for Efficient Modeling of Reconfigurable Metasurfaces in RIS Devices

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This study introduces lumped elements into a fast three-dimensional numerical solver based on an electric field integral equation (EFIE) formulation. The efficient implementation of lumped elements is essential for the rapid analysis of reconfigurable metasurfaces (MTSs), one of the most promising technologies for the realization of reconfigurable intelligent surfaces (RISs). The lumped elements are modeled by a super element constructed as a linear combination of Rao-Wilton-Glisson (RWG) functions. The extended solver is capable of the fast re-computation of the problem for different impedance values, thus, enabling numerous evaluations of the field in optimization loops, for example. The implementation of the lumped elements are validated and run-time data are presented.

Index Terms—Integral equation, method of moments, metasurface, reconfigurable intelligent surface

I. BACKGROUND

THE paradigm of reconfigurable intelligent surfaces (RISs) is a prominent architectural platform among the key enablers of the forthcoming sixth generation (6G) wireless networks [1]. Numerous RIS prototypes have been developed based on reconfigurable metasurfaces (MTSs) [2]. MTS technology opens a plethora of electromagnetic (EM) field transformations owing to the sub-wavelength conductor patterns exhibiting a fine-tunable homogenized boundary condition [1]. The accurate and fast modeling of the interaction between the impinging EM field and the MTS constitutes a relevant research question.

Utilizing electric field integral equations (EFIE) is one possible approach to address the problem at hand. One can find the overview of such techniques in [3]. The authors of the present paper have developed an efficient EFIE framework based on characteristic cell functions (CCFs) capable of modeling three-dimensional (3-D) problems. A vital difference of our approach compared to [3], is the capability to handle 3-D problems, while requiring the discretization of the MTS pattern only. The development of the framework is reported in four papers, a recent overview is available in [4]. In a nutshell the novelty of our approach is twofold. First, a spatial spectral method is utilized to numerically obtain the EM field of the basis functions. Second, CCFs are employed as global basis functions.

In this paper, we extend our framework with the formulation of lumped elements (LEs) to analyze reconfigurable MTSs. In the literature, there are different approaches for embedding LEs into integral equation frameworks [5]. As Gillard et.al. [5] explained, numerical issues arise when zero-length LE is assumed. Our approach is different from the solutions suggested in [5]; we construct the LE as a linear combination of the Rao-Wilton-Glisson (RWG) basis functions [6], forming a super element. The key feature of our extended EFIE framework is the ability to recompute the solution for tuned impedance values quickly. This feature makes the framework suitable

for large-scale evaluations, for example, in optimization loops where the proper value of multiple LEs is to be found. We present the verification of the LE formulation here, and we are going to analyze RIS benchmark problems in the full version.

II. FORMULATION

A. EFIE Solution

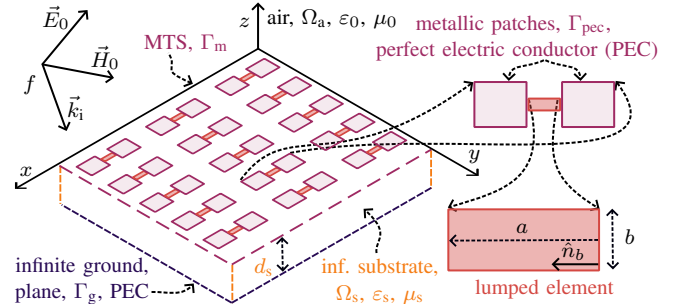


Fig. 1. The considered RIS-MTS arrangement in plane-stratified media. The inset shows the MTS cell used for validation.

The scenario we considered is depicted in Fig. 1. The formulation has been rigorously introduced in [4] so let us recall only a brief summary in order to present the LE implementation. We assume time harmonic EM fields and plane wave (PW) excitation. The surface EFIE can be formulated as

$$\vec{E}_t^p + \mathcal{G}_t \vec{K} = 0 \text{ on } \Gamma_{\text{PEC}}, \quad (1)$$

where $\vec{E}_t^p$ is the tangential component of the total electric field ($\vec{E}$) in a scenario without the MTS, and $\mathcal{G}_t$ is the linear operator providing the electric field of the induced surface current density ($\vec{K}$). Then, we apply a complete set of expansion functions ($\vec{\beta}_n$) to discretize $\vec{K}$ as

$$\vec{K} \approx \sum_{n=1}^N K_n \vec{\beta}_n, \quad (2)$$

where K_n are unknown coefficients, $\vec{\beta}_n$ can be for example the RWG expansion functions ($\vec{\Lambda}_n$) or the set of CCFs [4]. Plugging (2) into (1), and following the standard Galerkin procedure, one obtains a linear system consisting N equations, yielding the K_n coefficients.

B. Lumped Elements

Let us consider a single LE here, but the procedure can be straightforwardly extended for arbitrary many. We expand the surface current density on the LE by the RWG functions as

$$\vec{K}_L = \sum_{m=1}^M c_m \vec{\Lambda}_m, \quad (3)$$

where the c_m coefficients are determined for a given LE geometry such that $\vec{K}_L$ is homogeneous within the LE, utilizing the assumption that the LE is small concerning the wavelength. Subsequently, $\vec{K}_L$ is also added to the set of expansion functions. The current flowing through the LE is

$$i_L = \int_b K_{N+1} \vec{K}_L \hat{n}_b dl, \quad (4)$$

where K_{N+1} is its unknown coefficient, b and $\hat{n}$ are introduced in Fig. 1. Recalling (1), the voltage drop on the lumped element is

$$u_L = z_L i_L = \int_a \vec{E}_t^p + \mathcal{G}_t \left(\sum_{n=1}^N K_n \vec{\beta}_n + K_{N+1} \vec{K}_L \right) d\vec{l}, \quad (5)$$

where z_L is the impedance of the lumped element, a is defined in Fig. 1 and $d\vec{l}$ refers to the line integral of the vector field. Plugging the extended $\vec{K}$ – in the brackets in (5) – into (1), and applying the Galerkin procedure – without testing with $\vec{K}_L$ –, we have a set of N equations with $N + 1$ unknowns. The last required equation is constructed via substituting (4) into (5). Let us emphasize that the change of z_L effects one coefficient in the extended linear system, enabling the fast recomputation of the problem for different z_L values.

III. VALIDATION

We verify our LE implementation by calculating the field of a single MTS cell consisting of two 70 mm $\times$ 50 mm sized rectangular PEC patches. There is a 10 mm wide gap between the patches in the x direction. The LE connecting them is 10 mm $\times$ 5 mm. The parameters of the substrate are $\varepsilon_s = 3.666\varepsilon_0$, $\mu_s = \mu_0$, $d_s = 1.524$ mm. The structure is illuminated by a PW having $f = 3$ GHz, $\vec{E}_0 = 1$ V/m $\hat{x}$, and $\vec{k}_i = -\hat{z}$. 280 and 4 isosceles right triangles are used to mesh each patch and the LE, respectively. Then, RWGs are applied. First, $z_L = -53.052j \Omega$ (1 pF @ f) is set. Having the K_n coefficients computed, the electric field can be efficiently evaluated on an arbitrary plane with our framework in a post-processing step. For $z = 20$ mm the result is shown in Fig. 2. Then, we solve the problem again for different z_L values. However, this time, with the aid of the available linear system. Subsequently, we compare the total electric field against a reference obtained by Ansys High-Frequency Structure Simulator (HFSS) [7]. The

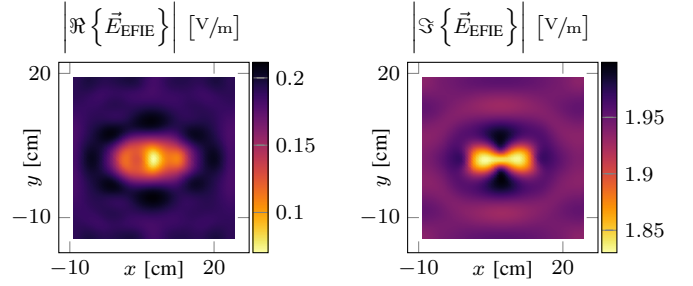


Fig. 2. $\vec{E}$ based on EFIE in the $z = 20$ mm plane for $z_L = -53.052j \Omega$

TABLE I
NORMALIZED GLOBAL RMS ERROR OF $\vec{E}_{\text{EFIE}}$ AGAINST HFSS

z_L [Ω]	Error [%]	z_L [Ω]	Error [%]
0	1.735	$-530.516j$ (0.1 pF)	1.628
1	1.732	$-53.052j$ (1 pF)	2.018
10	1.723	$-5.305j$ (10 pF)	1.774
100	1.763	$-0.531j$ (0.1 nF)	1.74
1000	1.392	$-0.053j$ (1 nF)	1.736

comparison in the $z = 20$ mm plane is given in Table I. The normalized global root-mean-square (RMS) error of the electric field is below the 5 % target precision for all the considered z_L values. Finally, let us report the RAM usage and computational time. A PC with i7-4930K and 64 GB RAM is used. The HFSS solution uses 49 GB and 9 min. The initial solution with our EFIE framework requires 5 GB and 78 min. Every calculation for a new z_L value takes only 2 s and 130 MB, which is a significant reduction. As we expect from the nature of the underlying numerical techniques, the number of unknowns are 3 651 498 for HFSS and 793 for our Python code.

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