

Dot Sensitivity Analysis for Current Source Reconstruction

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This paper proposes a method for reconstructing current sources with reduced computational cost. By employing current dot sensitivity analysis, the method eliminates the need for iterative finite element analysis (FEA) typically required in conventional approaches and enables direct current source design. To demonstrate the efficiency and robustness of the proposed method, a numerical example was conducted to non-invasively reconstruct current sources from magnetic flux density data.

Index Terms—Inverse problems, reconstruction algorithms, sensitivity analysis, and system identification.

I. INTRODUCTION

INVERSE PROBLEM in magnetostatic systems has been focused in various applications. The current source reconstruction problem is an indispensable topic to be explored in areas such as the non-invasive fault detection in fuel cells and the visualization of currents inside a body of biomagnetic imaging. Since these problems have cumbersome properties of ill-posed data and underdetermined solutions, diverse algorithms and techniques have been developed. When identifying current source distributions non-intrusively, methods typically rely on the physical relationships between reference data and sources. The methods frequently employ discretized models so that data from various source configurations are linearly combined to compute the solution [1]. This approach is advantageous as it derives solutions with a small amount of data and reduced computational time. However, it rapidly increases computational cost as the discretization becomes finer. To identify shape and topology of current sources, continuum sensitivity analysis and level-set techniques were employed in prior optimization studies [2]. These optimization methods enabled solution derivation with fast convergence, regardless of the dimensions of the discrete model or design variables. The methods also used the finite element method (FEM) with high accuracy and credibility to analyze the magnetostatic system. Nevertheless, these methods often hinge on the initial shape and are prone to converging to local optimum solutions. Since they also demand an iterative optimization process, the robust results increased number of FEM analyses, which extended the optimization time.

In this paper, we propose an approach that addresses the limitations of foregoing methods while effectively utilizing their strengths to solve inverse source problems. The key aspect of the proposed method is reconstructing the current distribution using the dot sensitivity analysis. The dot sensitivity is defined as the variation in the objective function caused by the creation of a current dot which falls under the continuum topology sensitivity. By utilizing dot sensitivity analysis, the information required for source reconstruction can be obtained with a single execution of FEA. Moreover, this method globally estimates the influence of the designed current distribution on the objective function by discretizing design domain. As a

result, it does not require an initial shape and significantly reduces the likelihood of converging to local optimum. The detailed process and assumptions are described in the main part.

II. CURRENT DOT SENSITIVITY ANALYSIS

Current dot sensitivity is defined as the variation of the objective function when the current dot is created in the base region. A general objective function is expressed as an integral of a continuously differentiable function g involving the magnetic vector potential $\mathbf{A}$ and the magnetic flux density $\mathbf{B}(\mathbf{A})$:

$$F = \int_{\Omega} g(\mathbf{A}, \mathbf{B}(\mathbf{A})) m_p d\Omega \quad (1)$$

where m_p is a characteristic function of the integral domain Ω_p . The current dot sensitivity $D(\mathbf{x})$ for this objective function was derived based on the continuum shape sensitivity in a previous study [3]:

$$D(\mathbf{x}) = \mathbf{J}_d \cdot \boldsymbol{\lambda} \quad (2)$$

where $\mathbf{J}_d$ is a current dot density and $\boldsymbol{\lambda}$ is an adjoint variable which was introduced when deriving the continuum shape sensitivity. The $\mathbf{A}$ and the $\boldsymbol{\lambda}$ were obtained as solutions of the magnetostatic governing equation and the adjoint equation, respectively:

$$\nabla \times \nu \nabla \times \mathbf{A} = \mathbf{J} \quad (3)$$

$$\nabla \times \nu \nabla \times \boldsymbol{\lambda} = -(\mathbf{g}_A + \nabla \times \mathbf{g}_B) m_p \quad (4)$$

where $\mathbf{J}$ is a current density source, and denotations of $\mathbf{A}$ and $\mathbf{B}(\mathbf{A})$ mean the partial derivatives of the function g with respect to $\mathbf{A}$ and $\mathbf{B}(\mathbf{A})$, individually. When solving (3) and (4), FEM was used with COMSOL Multiphysics software. Based on (2), the dot sensitivity analysis was performed in the numerical modeling to reconstruct the current distribution.

III. NUMERICAL MODELING FOR CURRENT RECONSTRUCTION

Numerical modeling was constructed to perform current reconstruction using dot sensitivity analysis. The objective function was formulated as follows:

$$F = \int_{\Omega_p} (\mathbf{B} - \mathbf{B}_0)^2 m_p d\Omega \quad (5)$$

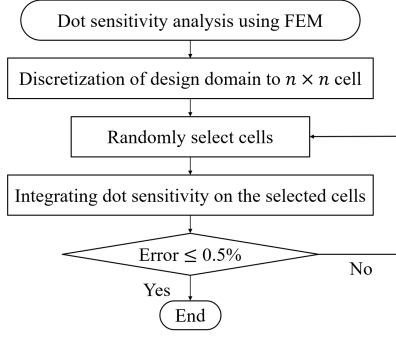


Fig. 1. Flowchart for the current reconstruction process.

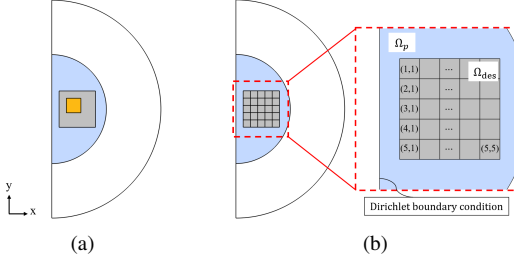


Fig. 2. Numerical modeling (a) reference model (b) initial model.

where $\mathbf{B}_0$ is the magnetic flux density from the reference model and $\mathbf{B}$ is from the initial model within Ω_p

In the initial model, $\mathbf{B}$ had a zero field distribution since it was vacant. The source term of (4) was determined based on (5) as follows.

$$\nabla \times \nu \nabla \times \boldsymbol{\lambda} = -2\nabla \times (\mathbf{B} - \mathbf{B}_0)m_p \quad (6)$$

The variables obtained through FEM were used to compute (5) and (2) in the current design domain Ω_{des} . The current density of the current dot was provided as prior information.

In this model, the optimal current distribution was derived with the process illustrated in Fig. 1. This process relied on the fact that the reduction of the objective function, when current dots are created precisely in the area corresponding to the reference current area, is equal to the calculated value of (5). The design domain was first discretized into an $n \times n$ grid of cells. A subset of these n^2 cells was randomly selected. The (2) was then integrated for the area of selected cells which is denoted as cell sensitivity. If the percentage error, the difference between the cell sensitivity value and the objective function value of (5) divided by (5), exceeded 0.5%, the cells were reselected.

$$\text{error} = \frac{\text{cell sensitivity} - (5)}{(5)} \times 100 [\%] \quad (7)$$

This process was concluded until (7) became lower than 0.5%, deriving the final current distribution.

IV. NUMERICAL EXAMPLE AND RESULT

To demonstrate the effectiveness of the proposed method, a numerical example was conducted to reconstruct a simple rectangular cross section current distribution. Fig.2 illustrates

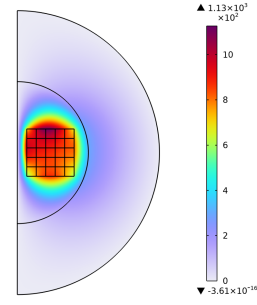


Fig. 3. Results of dot sensitivity analysis.

TABLE I
RESULTS FOR CELL SENSITIVITY AND PERCENTAGE ERROR

Indices of Selected 4 Cells	Cell Sensitivity [MA*Wb/m]	Error [%]
(2,2) (2,3) (3,1) (5,4)	1.441	1.27
(2,2) (2,3) (3,1) (5,5)	1.400	4.06
(2,2) (2,3) (3,2) (3,3)	1.459	0.00
(2,2) (2,3) (3,2) (3,4)	1.447	0.87

the reference model to be identified and the initial model to be designed. The Ω_{des} was assigned to a rectangular region encompassing the reference current and the Ω_p was set as a half-circle region surrounding the Ω_{des} . The design domain of the initial model was discretized into a 5×5 grid of cells. For clarity, each cell was sequentially indexed from (1,1) in the upper left corner to (5,5) in the lower right corner, as shown in Fig. 2. The reference current was assigned to the cells with indices (2,2), (2,3), (3,2) and (3,3). The current density was uniformly distributed at 2×10^6 A/m². The homogeneous Dirichlet boundary condition was imposed on the plane of symmetry. The proposed method was applied to this example. Fig. 3 presents the results of the dot sensitivity analysis. Table 1 shows selected cases of cell sensitivity and percentage error where only four cells were chosen. The calculated value of (5) was 1.459×10^6 . When the cells corresponding to the reference current distribution were selected, the percentage error approached zero, validating the accuracy of the proposed approach. This result proves that applying dot sensitivity analysis to the inverse problem allowed to obtain the solution with only a single FEA execution, eliminating the need for iterative procedures. Additionally, employing techniques such as genetic algorithms or evolutionary strategies for cell selection could further improve the efficiency of the solution process.

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