

Grey-Box Modeling of a Variable Flux Reluctance Machine Towards Scalable Electromagnetic Design

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In reluctance machines, optimizing electromagnetic designs becomes more demanding with repeated evaluations of complex topologies for geometric parameter scaling. This paper presents a grey-box magnetic modeling approach for the electromagnetic scaling of variable flux reluctance machines (VFRMs). The proposed model integrates reduced-order finite element model (FEM) data into a semi-analytical harmonic model (HM) with mode-matching. This grey-box magnetic model combines the geometry-scaling flexibility and computational efficiency of HM while accounting for magnetic saturation effects with FEM data. Gaussian process regression (GPR) is employed to create a reduced-order data merging via Fourier series (FS) coefficients of the airgap magnetic flux density. The resulting regression model serves as a post-processing tool for HM applied to various scaled geometries.

Index Terms—Electromagnetic modeling, electromechanical systems, Gaussian processes, harmonic analysis

I. INTRODUCTION

THE SHIFT towards rare-earth-free electrical drives in traction applications highlights the advantages of variable flux reluctance machines (VFRMs) as a low-cost, robust alternative to permanent magnet (PM) machines, offering higher power and torque density and extended field-weakening capability. Despite these benefits, the doubly-excited structure of VFRMs demands high currents, leading to challenges such as self- and cross-magnetic saturation effects [1]. The optimization of such reluctance machines typically includes scaling the electromagnetic parameters. These optimization processes involve analyzing and adjusting dozens of the machines' internal geometric parameters. Thus, the scaling procedure requires evaluating the electromagnetic models numerous times to obtain the performance parameters. Finite element models (FEM) offer high accuracy but are computationally expensive, while semi-analytical methods like harmonic modeling (HM) are faster but less accurate, particularly under saturation.

This paper proposes a hybrid method, a grey-box magnetic model that enhances HM with FEM-based dataset to include the saturated machine characteristics, since the machine mainly operates near that regime, as mentioned previously. In (semi-) analytical electromagnetic modeling methods, nonlinear characteristics of the ferromagnetic material are usually incorporated into the solution through iterative solvers [2]. Differently, the grey-box modeling approach achieves this holistically. The method relies on first obtaining the airgap magnetic field distribution for different current excitation values, and by varying a set of limited number of geometrical parameters that would cause saturation in the ferromagnetic parts of the machine, such as rotor/stator tooth width and back iron thickness. Then, the relationship between those parameters, which are current values and geometric parameters, and the difference between Fourier series (FS) coefficients of the airgap flux density obtained using the HM and FEM, is determined using Gaussian process regression (GPR). GPR is particularly advantageous in

this context due to its generalized nature and non-parametric framework. In the final step of the grey-box model, the resultant regression model is used to train the FS coefficient data (of the airgap flux density) of an arbitrary design scaled with HM. The paper first introduces the HM technique in a general sense and shows the results from an example of 12 stator and 10 rotor pole VFRM topology. Afterwards, the grey-box modeling method and the formulation of the GPR in this context are presented.

II. HARMONIC MODELING (HM) METHOD WITH MODE-MATCHING

The HM method relies on obtaining the magnetic flux density ($\vec{B}$) distribution from magnetic vector potential ($\vec{A}$) using the magnetoquasistatic approximation of Maxwell's equations [3]. For a certain region of the geometry, Poisson's or Laplace's equation provides the relationship between these magnetic quantities, depending on the presence of a field source.

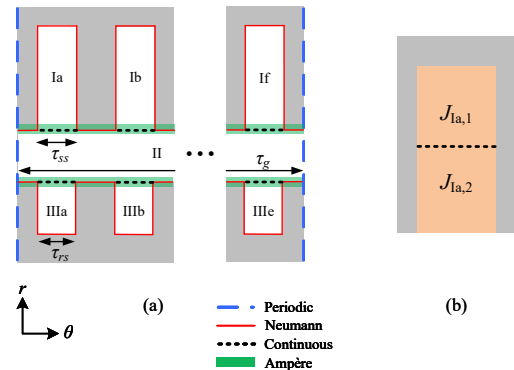


Fig. 1. Modeled VFRM geometry in polar coordinates: (a) Description of regions and boundary conditions, (b) Placement of the current source terms within the stator slot.

Fig. 1 shows the modeled VFRM geometry in a polar coordinate system. For the 12 stator 10 rotor pole topology, a

half symmetry is considered. In the given geometry, regions Ia-Ib denote the six stator slots. For these regions which include current sources (see Fig. 1b), Poisson's equation with magnetoquasistatic approximation can be written as:

$$\nabla^2 \vec{A}^k = \nabla \times \vec{B}^k = -\mu_0 \vec{J}^k, \quad (1)$$

where the superscript k denotes the region number of each term. For regions II and IIIa-IIIe, which stand for the airgap and five rotor slots, respectively, Laplace's equation with magnetoquasistatic approximation can be used as in (2):

$$\nabla^2 \vec{A}^k = \nabla \times \vec{B}^k = 0. \quad (2)$$

The radial (r) and tangential (θ) components of $\vec{B}$ can be expressed as a FS consisting of N harmonics:

$$B_r^k(r, \theta^k) = \sum_{n=1}^N [B_{rsn}^k \sin(w_n^k \theta^k) + B_{rcn}^k \cos(w_n^k \theta^k)],$$

$$B_\theta^k(r, \theta^k) = \sum_{n=1}^N [B_{\theta sn}^k \sin(w_n^k \theta^k) + B_{\theta cn}^k \cos(w_n^k \theta^k)] + B_{\theta 0}, \quad (3)$$

where w_n^k is the spatial frequency of each region, denoted by k . w_n^k can be expressed in terms of τ_k , the width of each region, shown in Fig. 1a, as $2\pi n/\tau_k$.

The FS coefficients in (3) can be found using boundary conditions (BC), illustrated in Fig. 1a. Assuming the stator and rotor steel to be magnetically linear and infinitely permeable, Neumann BCs can be applied at a given radius, R_0 , which leads to:

$$B_\theta^k = 0|_{r=R_0}. \quad (4)$$

For the boundaries between slot openings and the airgap, continuous BCs are applied at a given radius, R_0 :

$$B_r^k = B_r^{\text{II}}|_{r=R_0}. \quad (5)$$

Finally, to ensure the conservation of magnetic flux around the airgap, which is implied by Ampere's law, the harmonics of the flux densities for different regions in (4) and (5) are equated through correlation functions since the regions have different spatial frequencies, namely, mode-matching. Using (4) and (5), a system of linear equations can be obtained, and its solution provides the FS coefficients in (3). Using the coefficients, the flux density distribution can be computed for the air regions.

Fig. 2 shows the flux density distribution in the middle of the airgap for a dc-current (I_{dc}) and ac-current (I_{ac}) excitation of 10 A, comparing the HM with a FEM model while considering linear ferromagnetic material characteristics. The results are in accordance, but for high values of current excitation, the HM (and the linear FEM model) loses its reliability because of the saturation of the steel.

III. GREY-BOX MODELING METHOD

By definition, a grey-box model is the combination of a physics-based (white-box), and a data-driven statistical (black-box) model. The strategy considers the HM to be the white-box part, and the black-box part is a GPR model based on the FEM data. The FEM data is generated by varying a set of parameters ($\bar{x}$) that affect the magnetic saturation, such as I_{ac} ,

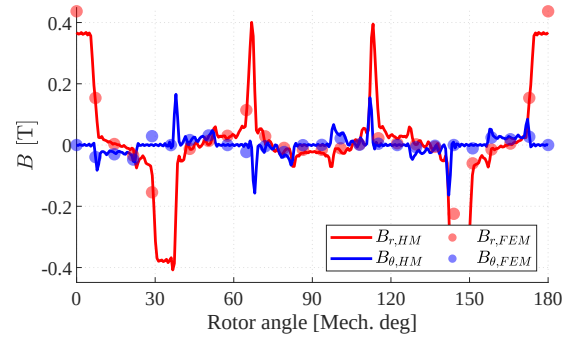


Fig. 2. Radial and tangential B distribution in the middle of the airgap of the VFRM for $I_{ac} = 10$ A and $I_{dc} = 10$ A: a comparison of HM with FEM.

I_{dc} , and tooth dimensions, where the main output parameter ($f(\bar{x})$) is the FS coefficients of the airgap $\vec{B}$, in the form of (3). The difference between the FEM-based and HM-based FS coefficients are then modeled using GPR.

Given a set of training points, a Gaussian process (GP) shows the function set that those training points represent [4]. For our dataset which ignores the measurement noise, a GP can be defined as the conditional probability between a given prior distribution, which is the FEM data, and the posterior predictions as:

$$\begin{bmatrix} f(\bar{x}) \\ f(\bar{x}_*) \end{bmatrix} \sim \mathcal{N} \left(\begin{bmatrix} m(\bar{x}) \\ m(\bar{x}_*) \end{bmatrix}, \begin{bmatrix} K(\bar{x}, \bar{x}) & K(\bar{x}, \bar{x}_*) \\ K(\bar{x}_*, \bar{x}) & K(\bar{x}_*, \bar{x}_*) \end{bmatrix} \right), \quad (6)$$

where $*$ denotes the prediction points and values, m denotes the mean, and K represents the covariances. GPR models the function covered by the training dataset by minimizing point covariances, achieved through hyperparameter optimization to maximize the GP model's likelihood.

IV. FUTURE WORK

This paper presents a grey-box magnetic modeling method towards a scalable design tool for VFRM optimization, which can also be extended to other machine topologies, combining a semi-analytical harmonic model with FEM data. By tuning the FS coefficients of the airgap magnetic field using GPR, the proposed approach enables computationally efficient geometry scaling while preserving the effects of magnetic saturation. In the full version, the FEM data generation method, the implementation details of the grey-box model will be discussed in detail, and the results from scaled models will be presented.

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