

# Performance of Newton-Raphson Method with A-Formulation for Superconductor Applications

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The performance of the Newton-Raphson method with A-formulation for superconductor applications is examined. While the Finite Element Method with A-formulation has been one of the popular choices in today's electrical designs, there is an argument that the combination of the A-formulation and the Newton-Raphson method has a convergence issue, especially for problems that consist of superconductors and normal magnetic materials. Such a problem was solved with A-formulation. The results showed that although a bare Newton-Raphson method suffered from iteration deterioration, it was recovered by introducing a relaxation technique and showed enough performance for complex problems.

**Index Terms**—Finite element analysis, Superconductor, Newton-Raphson method, A-formulation

## I. INTRODUCTION

SUPERCONDUCTOR TECHNOLOGY is a key technology for several applications where high magnetic flux density is necessary and high energy-saving is desired. Nuclear fusions and superconducting generators are recent concrete examples. While the superconductor is capable of having high current density with little loss compared to normal conductors, the property is guaranteed only with limited conditions that are sensitive to conditions such as temperature, stress, and magnetic flux density. The design of devices using superconducting materials has to ensure that the conditions are maintained.

Numerical simulations, such as finite element analysis (FEA), is an ideal tool for examining designs and observing physical situations in superconducting materials. The paper [1] describes the hysteresis loss calculation by FEA of double-stranded superconducting cables. Since the superconductor simulation by FEA is basically an eddy current calculation with nonlinear electric conductivity, the simulations tend to become large due to fine mesh to represent spatial profiles of the shield currents. So, further complex problems, such as assemble level problems, are difficult to solve with FEA. To overcome the difficulty, PEEC was proposed, and the assembly level problems were nicely solved [2].

Nevertheless, FEA has been commonly used for electrical machine designs in a wide range of applications and is expected to be continuously used for complex superconductor applications including the magnetic materials and normal conductors. Regarding digesting large-size problems, some effective techniques have been developed to shorten the calculation time, and these problems can be solved in a practical timeframe today [3].

There are several types of formulations of FEM. The differences come from the choice of variables to be solved. Common choices are A-formulation with vector potential  $A$  and H-formulation with magnetic field  $H$ , among others. In the context of the superconductor simulation, there is an argument about the choice of the formulation.

The paper [4] proposes a coupled formulation of H- and A-formulation where H-formulation and A-formulation are applied to the superconductor region and non-superconductor region, respectively. The method can enjoy the best efficiency in non-linear iterations for superconductor and normal magnetic materials. By taking the coupled approach, the Newton-Raphson method (NR), considered more effective than the Picard iteration method, can be naturally used for both regions. This implies that dealing with the whole region through A-formulation is not a practical choice because the convergence of NR deteriorates.

On the other hand, A-formulation is a popular choice for today's machine design and is desired for superconductor applications as well, inheriting accumulated knowledge and experience. Therefore, it is important to evaluate the performance of NR with A-formulation for superconductor applications is investigated, which is reported in this paper.

## II. FORMULATION

The magnetic field of a region including superconductors (SC) can be described by using A-formulation where current density  $\mathbf{J}$  and electric field  $\mathbf{E}$  are connected with electric conductivity as eq.(1).

$$\mathbf{J} = \sigma \mathbf{E} \quad (1)$$

The electric conductivity of SC is represented by the N-value-model as eq.(2).

$$\sigma = J_c(|\mathbf{B}|) \frac{1}{E_c} \left( \frac{|\mathbf{E}|}{E_c} \right)^{\frac{1}{N}-1} \quad (2)$$

where  $\mathbf{B}$ ,  $J_c$ ,  $E_c$ , and  $N$  are magnetic flux density, the critical current density, threshold electric field defining the critical current density, and a parameter characterizing the function shape, respectively. Since  $\sigma$  depends on  $\mathbf{E}$  and  $\mathbf{B}$ , the eq.(1) becomes nonlinear and needs to be solved in an iteration manner with either NR or the Picard iteration method.

### III. PERFORMANCE OF NEWTON-RAPHSON METHOD

#### A. Superconductor only

The performance of NR for systems which have SC only as nonlinearity is examined. The model has two coils each of which consists of 100 layers of REBCO tape. The REBCO tape has  $J_c=6.4e+8A/mm^2$ . The value of  $N$  varies from 4 to 40. Two versions of NR, with relaxation and without, are tested.

The results are summarized in Table 1, which shows that the NR without the relaxation fails to converge except for the case  $N=4$ , which has the weakest nonlinearity. The NR with the relaxation shows reasonable convergences for all cases. The current distribution in the case of  $N=17.86$  is shown in Fig.1.

#### B. Superconductor and ferromagnet

The paper [4] suggests that systems that contain both the SC nonlinearity and the magnetic nonlinearity cause poor convergence when only A- or H-formulation is used, by showing the calculations by H-formulation with NR and by A-formulation with the Picard iteration method.

We calculated the same model using the A-formulation and NR. The results are shown and compared with the reference [4] which are carried out by coupled H-A-formulation, in Table 2. It is found that, as in the above example, NR with the relaxation [5] achieves convergence with reasonable iterations, which are close to the numbers by the coupled H-A-formulation.

### IV. ASSEMBLY LEVEL PROBLEM

An assembly level problem [2] is solved to examine the performance of A-formulation using NR with the relaxation method. The model is shown in Fig.2. The basic structure and material are the same as the example in III-A. However, this model has further details, such as spiral structure and lead wires, and the entire coil is modeled to take into account the asymmetry which the details introduce. The 3D FEA model has 2,620,129 elements and the number of time steps is 40. The value of  $N$  is 17.86.

As a result, the I-V relationship is shown in Fig.3, which shows reasonable agreements with the measurements shown in the reference [2]. The calculation time was 6.4 hours using 16 cores of CUP Intel Xeon Gold 6130 2.1 GHz with 32GB of memory.

### V. CONCLUSION

The performance of the Newton-Raphson method with A-formulation for superconductor applications was investigated.

A mixed superconductor and ferromagnet system was simulated. The results confirmed that although the bare Newton-Raphson method showed deterioration in convergence, it was recovered and showed the same level of convergence performance as the coupled H-A-formulation by employing a relaxation technique. The calculation scheme was also applied to an assembly level problem which has a complex structure and has over 2.5 million elements. The problem was nicely solved without the convergence deterioration.

Further validation of A-formulation for complex superconductor applications is necessary and is our future work.

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| N-value | With relaxation factor | Without relaxation factor |
|---------|------------------------|---------------------------|
| 4       | 11                     | 46                        |
| 10      | 16                     | Not converged             |
| 17.86   | 21                     | Not converged             |
| 30      | 21                     | Not converged             |
| 40      | 22                     | Not converged             |

| Iteration method | Formulation |             | Coupled H-A |    | A      |       |
|------------------|-------------|-------------|-------------|----|--------|-------|
|                  | SC          | Ferromagnet | NR          | NR | Picard | NR    |
| Mesh             | Coarse      |             | 1,071       |    | 7,009  | 756   |
|                  | Medium      |             | 1,931       |    | 13,317 | 2,236 |
|                  | Fine        |             | 3,753       |    | 19,351 | 5,470 |

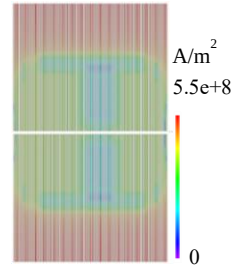


Fig. 1. Current density distribution of REBCO tapes

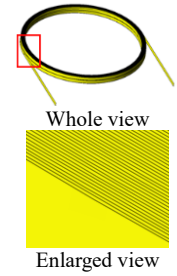


Fig. 2. Assembly level model  
The spiral structure, tape layers, and lead wires are precisely modeled.

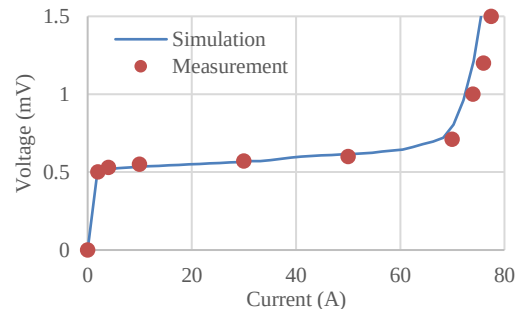


Fig. 3. I-V relationship