

FEM Based 2D Equivalent Modelling of Planar Inductors

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In this work, we present a novel Finite Element Method (FEM)-based approach for modeling planar inductors using an equivalent two-dimensional (2D) formulation. The proposed methodology aims to significantly reduce computational costs while maintaining high accuracy in electromagnetic analysis. Traditional full three-dimensional (3D) models, though precise, are often computationally expensive and may become impractical for iterative design or optimization tasks. By leveraging the inherent symmetries and geometric properties of planar inductors, we derive an equivalent 2D model that accurately represents the electromagnetic behavior of the system, including inductance, losses, and field distributions. The approach is validated through several case studies, comparing the 2D model's predictions against both full 3D simulations. Results demonstrate that the equivalent 2D model achieves computational savings of up to 90% while preserving an error margin below 5% in key performance metrics. This work provides a robust tool for the design and optimization of planar inductors, particularly in applications where computational efficiency is critical, such as in power electronics, RF systems, and wireless energy transfer.

Index Terms—Planar inductors, Parameter Extraction, 2D Models, Printed Circuit Board (PCB).

I. INTRODUCTION

Planar inductors are fundamental components in a wide range of applications, including power electronics, RF systems, and wireless energy transfer. Their accurate electromagnetic modeling is crucial for optimizing performance metrics such as inductance, quality factor, and thermal dissipation. Conventional three-dimensional (3D) modeling techniques, while accurate, are computationally expensive, especially when applied to iterative design or optimization tasks involving complex layouts [1]. Consequently, there is a growing demand for reduced-order modeling strategies that balance computational efficiency with high accuracy.

Among the techniques proposed for such purposes, two-dimensional (2D) or 2.5D methodologies have gained significant attention for electronic package and printed circuit board (PCB) modeling [2]. These approaches exploit the inherent planar geometry of multilayer structures to simplify computational requirements while preserving key electromagnetic phenomena. For example, modal decomposition methods in 2.5D modeling decouple complex multilayer problems into manageable subdomains, achieving a trade-off between computational efficiency and model fidelity. Similarly, hybrid strategies combining numerical and semi-analytical techniques have demonstrated the ability to capture both low-frequency and high-frequency effects without resorting to full-wave 3D simulations [3]–[5]. Other approaches based on Integral Equation have been proposed for modeling Printed Circuit Boards (PCB) [6], [7].

Inspired by these advancements, this work proposes a novel Finite Element Method (FEM)-based approach for modeling planar inductors. By deriving an equivalent 2D model, we aim to address the limitations of existing full-wave methods, which often require significant computational resources to discretize the entire 3D domain. Our method leverages the planar symmetry of inductors and applies tailored boundary conditions and meshing strategies to accurately replicate the

electromagnetic behavior observed in the full 3D context.

This work represents a preliminary investigation into the electromagnetic modeling of planar inductors using an equivalent 2D Finite Element Method (FEM) framework. For this initial study, we limit our focus to the stationary case (DC) to simplify the analysis and establish a foundation for future developments. In this context, we extract the equivalent resistance (R) and inductance (L) of the inductor while varying key geometrical parameters. Future investigations will extend this framework to incorporate frequency-domain effects, enabling the study of skin effects, eddy currents, and complex impedance behavior under varying operational frequencies.

The 2.5D method provides over 10× computational savings compared to 3D models while maintaining scalability for large systems. Its modular design supports advanced tasks like transient signal analysis and efficiently handles ECAD files in their native 2D format. Additionally, it allows flexible, layer-specific meshing and offers opportunities for future extensions in material and dynamic analyses.

II. FEM BASED EQUIVALENT 2D FORMULATION

In this study, we adopt a simplified two-dimensional (2D) framework to model the electromagnetic behavior of planar inductors in DC condition. The approach revolves around solving the conservation of charge and magnetic field equations on discrete planes $z = z_i$, where i represents the plane index. Below, we outline the formulation in detail.

A. Charge Conservation Equation

The electrical potential V_i at the plane $z = z_i$ satisfies the charge conservation equation:

$$-\nabla_2 \cdot (\sigma \nabla_2 V_i) + \text{source}(V_{i-}, V_{i+}) = 0 \quad (1)$$

where ∇_2 and $\nabla_2 \cdot$ are the 2D gradient and divergence operators, respectively. The term $\text{source}(V_{i-}, V_{i+})$ accounts for currents entering or leaving the plane and is defined as:

$$\text{source}(V_{i-}, V_{i+}) = -\sigma^+ \frac{V_{i+} - V_i}{z_{i+} - z_i} - \sigma^- \frac{V_i - V_{i-}}{z_i - z_{i-}}. \quad (2)$$

Here, σ^+ and σ^- represent the conductivity out of the plane. They equal the bulk conductivity $\sigma = \sigma(x, y)$ if the planes i and $i + 1$ (or $i - 1$) are in contact, or zero otherwise.

B. Current Density

The in-plane current density $\mathbf{J}_i$ is derived from the potential V_i as:

$$\mathbf{J}_i = -\sigma \nabla_2 V_i, \quad (3)$$

where $\mathbf{J}_i = (J_{x_i}, J_{y_i})$ are the current density components in the x and y directions, respectively.

C. Magnetic Field Equation

The magnetic field in the z direction, H_{z_i} , satisfies:

$$\nabla \times (\nabla \times \mathbf{H} - \mathbf{J}) = \mathbf{0}, \quad (4)$$

that for the specific case reduces to

$$\nabla \times \nabla \times (\cdot) = -\nabla_2 \cdot (\nabla_2(\cdot)) - \frac{d^2(\cdot)}{dz^2}. \quad (5)$$

Since, for simplicity, planes are considered equally spaced, we can write

$$\frac{d^2 H_{z_i}}{dz^2} = \text{DZ2}(H_{z_{i-}}, H_{z_i}, H_{z_{i+}}) = \frac{H_{z_{i+}} + H_{z_{i-}} - 2H_{z_i}}{\Delta z^2}. \quad (6)$$

Thus, we obtain

$$-\nabla_2 \cdot (\nabla_2 H_{z_i}) - \text{DZ2}(H_{z_{i-}}, H_{z_i}, H_{z_{i+}}) + \mathbf{u}_z \cdot \nabla \times \mathbf{J}_i = 0, \quad (7)$$

where $\nabla \times \mathbf{J}_i$ is evaluated moving the curl from $\mathbf{J}$ to the test function.

D. Extensions and Discussion

The above equations can be extended by incorporating a magnetic scalar potential and vector potential components to model in-plane magnetic fields. These enhancements allow for a more comprehensive description of the electromagnetic behavior and improved handling of material discontinuities.

Finally, preliminary results indicate that the placement of equivalent electric and magnetic layers plays a critical role in achieving accuracy. The final paper will provide a detailed presentation and analysis of these findings.

III. NUMERICAL RESULTS

In this section, we present the numerical results obtained from the 3D and 2.5D simulations conducted using COMSOL Multiphysics where the proposed 2.5D formulation has been implemented. The study evaluates the resistance R and inductance L as functions of the number of layers used for the equivalent 2.5D model of a planar inductor made of copper shown in Fig. 1. Results obtained from 3D and 2.5D models are compared.

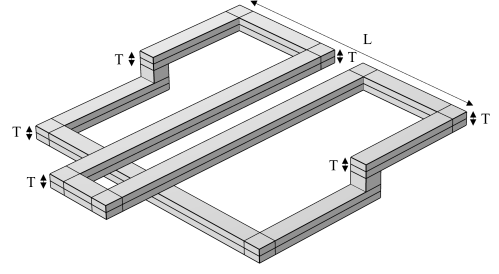


Fig. 1: 3D model used for the simulations. L is 1.6 mm and T is 25 μm .

TABLE I: Simulation results for resistance and inductance for the 2.5D and 3D models.

Model	L (nH)	R (m Ω)
3D	2.3145	57.027
2.5D	2.3318	57.033

The results are summarized in Table I, which compares the computed resistances and inductances for the 3D and 2.5D cases. The 3D and 2.5D results exhibit close agreement, with minor discrepancies attributed to model assumptions and approximations.

IV. CONCLUSIONS

This preliminary study introduces a 2.5D modeling approach for planar inductors, focusing on the stationary case. The proposed method demonstrates significant computational savings, scalability, and compatibility with ECAD workflows, making it a promising alternative to full 3D modeling. Future work will extend the approach to include frequency-domain effects and transient signal analyses.

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