

Estimation of Higher-Order Modes in Miter Bend Using Mode Decomposition Technique

Yoshihisa Fujita¹, Yuki Goto², and Shin Kubo³

¹College of Industrial Technology, Nihon University, Narashino, Chiba 275-8575, Japan, fujita.yoshihisa@nihon-u.ac.jp

²National Institute for Fusion Science, National Institutes of Natural Sciences, Toki, Gifu 509-5292, Japan, goto.yuki@nifs.ac.jp

³College of Science and Engineering, Chubu University, Kasugai, Aichi 487-8501, Japan, kuboshin@fsc.chubu.ac.jp

In this study, a mode expansion method for estimating specific eigenmodes from measurement data and simulation results was proposed. As a test case, the estimation of propagation modes in a miter bend, where the presence of higher-order modes is a challenge, was performed. The electromagnetic field distribution within the miter bend was calculated through numerical simulations, and the validity of the proposed method was verified by applying the mode expansion technique. By calculating the proportion of the propagation modes, it was demonstrated that the input mode propagates as the dominant mode. Additionally, the presence of higher-order modes was confirmed, and their specific proportions were successfully calculated.

Index Terms—Waveguide theory, Miter bend, Mode decomposition, Higher-order modes.

I. INTRODUCTION

HIGH-POWER millimeter waves are utilized in a wide range of fields, including communication[1], measurement[2], and heating[3]. Due to their high power, electromagnetic wave transmission is even more strongly influenced by the transmission environment. For this reason, waveguides have been primarily used for long-distance transmission. In particular, vacuum-sealed waveguides are employed for transmitting ultra-high-power millimeter waves used in nuclear fusion power generation[3]. Additionally, as waveguides with curvature alter the propagation distance, they are typically constructed using only straight and right-angle sections.

Corrugated waveguides are commonly used as transmission lines for high-power millimeter waves, as they are designed to minimize wall surface effects. When bending at right angles, miter bends are utilized. Miter bends allow for easy replacement of the reflective plates and are also employed in measurement and polarization control[4]. However, diffraction occurs near the mirrors, similar to free space, leading to the unintended generation of higher-order modes[5]. Since transmission lines are designed based on the assumption of fundamental modes, the occurrence of higher-order modes directly reduces transmission efficiency. Therefore, mechanisms to suppress higher-order modes are essential.

To control higher-order modes, it is first necessary to investigate the detailed composition of the propagating modes. General propagating modes are characterized by their radial and azimuthal orders. Recently, methods for the rapid estimation of topological charges, which correspond to the azimuthal mode order, have been proposed[6]. However, these methods do not account for radial information and have not achieved concrete mode estimation. In response, this study aims to propose a novel method for estimating specific propagation modes. Additionally, the proposed method will be applied to numerical simulation results to verify its effectiveness.

II. MODE DECOMPOSITION

In this study, the propagation direction of the waveguide is defined as the z -axis, and it is assumed that the Poynting flux of the waveguide mode in a cross-section is normalized as follows:

$$\int_0^{2\pi} \int_0^a (\mathbf{E} \times \mathbf{H}^*)_z r dr d\theta = 1, \quad (1)$$

where a denotes the inner radius of the waveguide, $\mathbf{E}$ is the electric field, $\mathbf{H}$ is the magnetic field, and the superscript asterisk represents the complex conjugate. Considering the coupling between different waveguide modes, the following orthogonality relation holds[7]:

$$\int_0^{2\pi} \int_0^a (\mathbf{E}_{mn}^\nu \times (\mathbf{H}_{m'n'}^{\nu'})^*)_z r dr d\theta = \delta_{mm'} \delta_{nn'} \delta_{\nu\nu'}, \quad (2)$$

where ν represents the type of propagation mode, m denotes the order in the azimuthal direction, and n represents the order in the radial direction. Additionally, the prime symbol signifies distinct variables. This orthogonality is demonstrated by the angular orthogonality:

$$\int_0^{2\pi} \frac{1}{2\pi} \exp(im\theta) \exp(in\theta)^* d\theta = \delta_{mn}. \quad (3)$$

and the radial orthogonality:

$$\int_0^a J_v \left(\frac{\chi_{vm}}{a} r \right) J_v \left(\frac{\chi_{vn}}{a} r \right) r dr = \frac{1}{2} a^2 [J_{v+1}(\chi_{vm})]^2 \delta_{mn}, \quad (4)$$

where J_m is the Bessel function of the first kind of order m , and its n th root is denoted as χ_{mn} . Moreover, from (3), it can be deduced that $\int_0^a J_m(\alpha r) J_n(\beta r) r dr$ can be neglected. (2) enable the estimation of the contribution of a specific waveguide mode within a propagating mode. By applying this method to all possible waveguide modes, the modal composition can be analyzed.

Since the coupling between waveguide modes is computed in the complex space, it cannot be directly applied to measurement data or simulation results obtained in the real space. Therefore, it is necessary to reconstruct the complex-space waves from the real data using Fourier transform. Given an electromagnetic field component f , its Fourier transform yields the frequency-domain function F as follows:

$$F(\omega) = \int_{-\infty}^{\infty} f(t) \exp(-i\omega t) dt. \quad (5)$$

Notably, F represents the complex amplitude and includes relative phase information. The inverse Fourier transform of F gives:

$$\tilde{f}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) \exp(-i\omega t) d\omega. \quad (6)$$

The tilde on $\tilde{f}$ indicates the electromagnetic field components in the complex space. Notably, since the frequency of the waveguide mode is constant regardless of its order, $\tilde{f}$ can be simplified as follows:

$$\tilde{f}(t) = |F(\omega)| \exp[i(\omega t - \arg F(\omega))]. \quad (7)$$

By applying this transformation to all electromagnetic field components, the complex-space waves can be reconstructed.

In this study, the orthogonality of Bessel functions is utilized under the assumption of waveguide modes. Alternatively, the orthogonality of Laguerre-Gaussian functions can be employed for mode decomposition in free space. Thus, this method can be considered a versatile approach for mode decomposition in electromagnetic field analysis.

III. NUMERICAL RESULTS AND DISCUSSION

First, to analyze the spatial distribution within the miter bend, numerical simulations were conducted. In this study, the finite-difference time-domain (FDTD) method was employed as the simulation technique[8]. The miter bend consists of two orthogonal cylindrical waveguides with a mirror placed at a 45-degree angle. The inner diameter of the waveguides is 4λ , the input waveguide length is λ , and the output waveguide length is 5λ . For computational efficiency, a cylindrical waveguide with smooth walls was assumed instead of the corrugated waveguide that is typically used. As a result, the eigenmodes are TM and TE modes; however, this study focuses exclusively on TM modes. In the simulation, it is assumed that the TM_{11} mode is incident from the negative z -axis, and the TM_{11} mode propagates out along the positive x -axis.

The electric field intensity obtained from the electromagnetic wave propagation simulation is shown in Fig. 1. In this study, all calculations are nondimensionalized with respect to the wavelength, and variables with an overbar represent the nondimensionalized quantities. Figures 1(b) and (c), representing the output and input cross-sections, respectively, reveal distortions in the field distribution. Furthermore, Fig. 1(c) demonstrates that the incident wave is reflected by the mirror. This indicates that, in addition to the intended waveguide mode, higher-order modes generated by diffraction are present. For reference, the analytical distribution of TM_{11} mode is shown in Fig. 1(d).

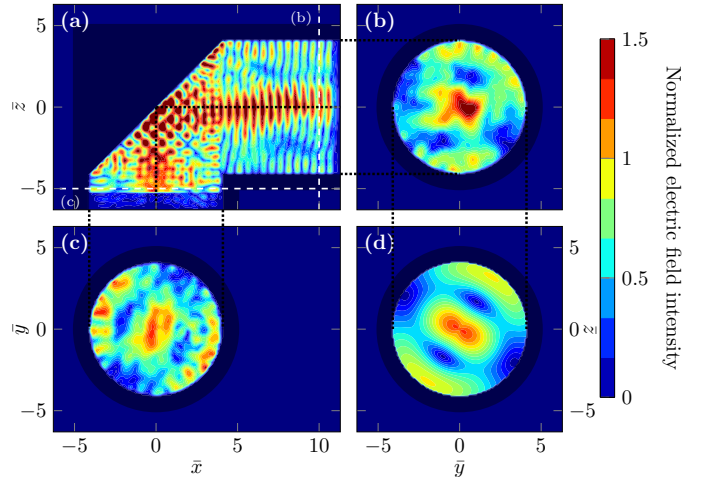


Fig. 1. Normalized electric field intensities $|\vec{E}|$ in the miter bend. (a) Cross section at $\bar{y} = 0$ parallel to optical axis. (b) Waveguide cross section at $\bar{x} = 10$. (c) $\bar{z} = -5$. (d) Analytical distribution of TM_{11} mode at $\bar{x} = 10$. The white dashed lines represent the cross-section, while the black dashed lines serve as auxiliary lines related to the axis and shape of the waveguide.

By comparing this figure with Fig. 1(b), you can see that the shading of the electric field intensity is roughly the same.

Subsequently, the proportion of the TM_{11} mode was estimated using the aforementioned formula, yielding 46%. Similarly, the proportions of the TM_{21} and TM_{31} modes were calculated as 6% and 10%, respectively. These results indicate that the dominant mode is the input mode, TM_{11} , and the contributions of other modes are minimal. This outcome is considered reasonable, demonstrating the validity of the proposed method. During the presentation, more detailed results, including those for other higher-order modes, will be discussed.

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