

Modeling of Radiation Wakefields using a Scattered Field Formulation

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Simulations of particle beam dynamics in electron accelerators need to account for both internal space charge effects and transient electromagnetic waves scattered at chamber walls. Commonly, these effects are treated separately using very different simulation techniques. We present a coupling procedure to account for both effects simultaneously by using a scattered field formulation. Here, we combine two field solvers optimized to simulate either the internal beam dynamics assuming free space or the transient electromagnetic wave propagation. Furthermore, in this paper we extend the approach to account for synchrotron radiation wakefields.

Index Terms—Computational electromagnetics, Electron accelerators, Electron beams, Synchrotron radiation,

I. INTRODUCTION

In the design process of electron accelerators, predicting the dynamics of the particle beam is crucial. While externally applied fields by magnets and RF cavities determine the macroscopic dynamics of a particle beam, internal space charge forces, synchrotron radiation (SR) and collective effects like wakefields can have a significant impact on the phase space distribution of the beam [1]. In principle, full electromagnetic Particle-in Cell (PIC) simulations can account for all these effects at the same time. However, the high computational costs associated with this approach makes them infeasible for real-case scenarios [2]. Hence, specialized models for the task of either space charge or wakefield simulations are used in beam dynamics simulations. Space charge solvers assume the bunch to move uniformly in free space, such that the particle-particle field can be obtained from an electrostatic problem in the bunch frame of reference [3]. Consequently, space charge simulations neglect field transients, synchrotron radiation as well as electromagnetic scattering by the surrounding geometry. Some approaches exist to incorporate scattered field contributions into space charge simulations in a postprocessing step [4]. It is furthermore possible to take into account radiation effects by evaluating the Liénard-Wiechert fields of the individual particles [5], [6]. On the other hand, synchrotron radiation also gets reflected at the vacuum chamber walls, exciting radiation wakefields. For this task, [7] developed a method to simulate these effects in time domain based on a pre-defined trajectory and by combining a global 2D mesh with a model expansion in the horizontal direction. Another approach starts from a frequency-domain description, also assuming a pre-defined trajectory [8]. We pursue a more generalistic approach by shifting the excitation of wakefields from the bunch current to an auxiliary magnetic surface current by means of a scattered field formulation.

II. SCATTERED FIELD FORMULATION FOR PARTICLE BEAMS

The electromagnetic wakefields of a particle bunch are described by Maxwell's wave equations in the time domain. In our wakefield computations, we employ the Finite Integration

Technique (FIT) to solve these equations [9]. In FIT, the degrees of freedom are electric and magnetic voltages on a staggered grid, denoted by $\hat{\mathbf{e}}$ and $\hat{\mathbf{h}}$, respectively. The FIT wave equations in semi-discrete (time-continuous, space-discrete) form then read as

$$\frac{d}{dt} \begin{pmatrix} \hat{\mathbf{e}} \\ \hat{\mathbf{h}} \end{pmatrix} = \begin{pmatrix} 0 & \mathbf{M}_\epsilon^{-1} \mathbf{C}^T \\ -\mathbf{M}_\mu^{-1} \mathbf{C} & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathbf{e}} \\ \hat{\mathbf{h}} \end{pmatrix} - \begin{pmatrix} \mathbf{M}_\epsilon^{-1} \hat{\mathbf{j}} \\ 0 \end{pmatrix} \quad (1)$$

with $\hat{\mathbf{j}}$ being the exciting current density of the particle bunch. The matrices $\mathbf{M}_\epsilon^{-1}$ and $\mathbf{M}_\mu^{-1}$ are the discrete material matrices and $\mathbf{C}$ is the topological curl operator. In the scattered field formulation, we decompose the total electric field in the wave equations into an incident field and a scattered field, $\mathbf{E}_t = \mathbf{E}_i + \mathbf{E}_s$, and accordingly for the magnetic field $\mathbf{B}$. We assume the incident field to be prescribed by the particle bunch and fulfilling Maxwell's equations in free-space. Then, on a perfectly conducting boundary (PEC) with normal vector $\mathbf{n}$, the scattered field needs to fulfill the boundary condition

$$\mathbf{n} \times \mathbf{E}_s|_{\text{PEC}} = -\mathbf{n} \times \mathbf{E}_i|_{\text{PEC}}. \quad (2)$$

In (1), this condition is included as an auxiliary magnetic surface current $\hat{\mathbf{j}}_m$, leading to the modified system of equations

$$\frac{d}{dt} \begin{pmatrix} \hat{\mathbf{e}}_s \\ \hat{\mathbf{h}}_s \end{pmatrix} = \begin{pmatrix} 0 & \mathbf{M}_\epsilon^{-1} \mathbf{C}^T \\ -\mathbf{M}_\mu^{-1} \mathbf{C} & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathbf{e}}_s \\ \hat{\mathbf{h}}_s \end{pmatrix} - \begin{pmatrix} 0 \\ \mathbf{M}_\mu^{-1} \hat{\mathbf{j}}_m \end{pmatrix}. \quad (3)$$

Here, the degrees of freedom $\hat{\mathbf{e}}_s$ and $\hat{\mathbf{h}}_s$ are, respectively, the electric and magnetic voltages of the scattered field on the grid. In the simple case of a staircase geometry approximation, given an incident field $\hat{\mathbf{e}}_i$ on the same grid as $\hat{\mathbf{e}}$, we can express the magnetic current as $\hat{\mathbf{j}}_m = \mathbf{C} \mathbf{I} \hat{\mathbf{e}}_i$ with a diagonal interpolation matrix $\mathbf{I}$ [10]. A far-field space charge solver provides the incident field $\mathbf{E}_i$ at the PEC, hence yielding $\hat{\mathbf{j}}_m$ in (3). The solution of (3) for the scattered field $\mathbf{E}_s$ is used to update the particle positions and momenta. The resulting charge density of the particles determines the space charge field of the beam. The near-field space charge field is computed by a grid-based 3D Green's function solver. For the far-field space charge field at the PEC boundaries, we use the mesh-free Fast Multipole Method (FMM) [12]. Fig. 1 highlights the different solvers involved as well as the moving computational window of the wakefield solver [13].

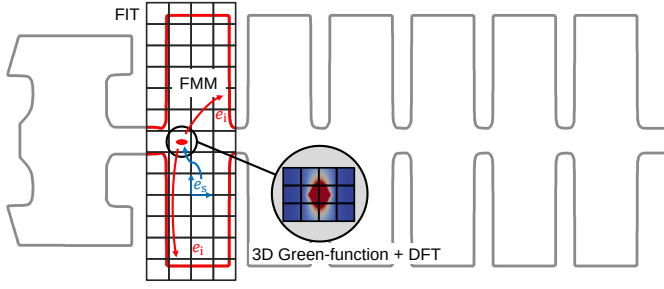


Fig. 1. Schematics of the coupling scheme between the space charge solver and the wakefield solver, including the FMM. The grey lines in the background depict an electron gun profile consisting of several RF accelerating cells [11].

III. ELECTRON GUN SIMULATIONS

To demonstrate the capabilities of our approach, we simulate an electron gun design taken from [11]. The electron gun features a series of RF cavities with very narrow iris apertures as shown in Fig. 1. By connecting every other cavity via side couplers, the resulting fields create a so-called quasi traveling wave field pattern. Together with the narrow iris aperture, RF focussing and strong acceleration can be obtained. However, geometric wakefields are generated at the narrow iris apertures, which can not be accounted for in conventional simulation tools. The results of beam dynamics simulations with and without wakefield effects using our coupled approach are shown in Fig. 2. It can be seen how the energy distribution along the bunch is altered by the wakefields, in this case opposing the energy chirp induced by the RF fields.

IV. SYNCHROTRON RADIATION WAKEFIELD

Using the FMM in the bunch rest frame to evaluate the far-field of the particle bunch neglects radiation and retardation effects. This assumption is justified in the electron gun due to the rapidly accelerating bunch and the linear motion. In a bunch compressor, the particle trajectory is curved (cf. Fig. 3) which leads to significant radiation effects [1]. To enable the simulations of such radiation wakefields, we extend our coupled simulation approach to consider the Liénard-Wiechert field of the accelerated particles as the incident field in the scattered field formulation (2). The electric field at position $\mathbf{x}$ and time t generated by a particle at retarded position $\mathbf{x}'$ and time t' is given by the well-known expression

$$\mathbf{E}(\mathbf{x}, t) = \frac{q}{4\pi\epsilon_0} \left[\frac{\mathbf{n}' - \beta'}{\gamma'^2 (1 - \mathbf{n}'\beta')^3 |\mathbf{x} - \mathbf{x}'|^2} + \frac{\mathbf{n}' \times [(\mathbf{n}' - \beta') \times \frac{\partial}{\partial t'} \beta']}{c (1 - \mathbf{n}'\beta')^3 |\mathbf{x} - \mathbf{x}'|} \right] \quad (4)$$

with the particle charge q , the speed of light c , the vacuum permittivity ϵ_0 , the Lorentz factor γ and the unit vector $\mathbf{n}'$ pointing from the particle to the evaluation point. As the far-field of the particle bunch is of interest for the incident field computation, only a small subset of the particles in the bunch needs to be considered. Combined with the usage of a moving window, the additional computational overhead of this approach is considered manageable.

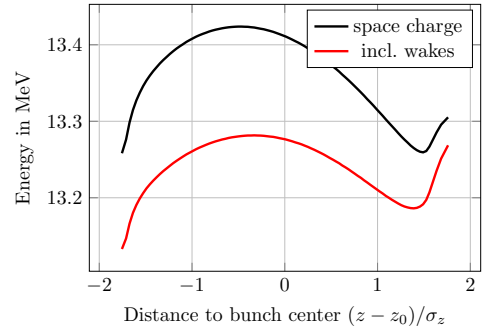


Fig. 2. Energy distribution along the electron bunch at the end of the gun.

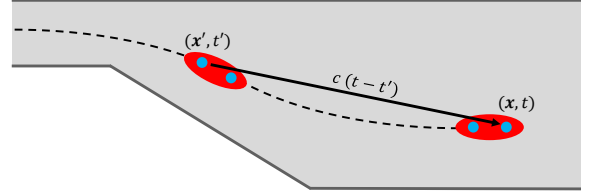


Fig. 3. Illustration of the curved trajectory in a bunch compressor and the radiation field seen at a position in the head of the bunch caused by a particle in the tail.

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