

A rapid calculation model for the loss of Litz wire of high-frequency transformers based on PGD and equivalent complex permeability

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This paper proposes a rapid calculation model for the loss of rectangular Litz wire high-frequency transformers based on the Proper Generalized Decomposition (PGD) spatial separation algorithm and equivalent complex permeability. A double 2D model is adopted to consider the difference of magnetic field distribution at the end windings of the inner and outer windows, in which the magnetic field is calculated by separating and reducing the two-dimensional finite element calculation into two one-dimensional finite element analysis using the PGD algorithm. Two different types of transformers are made and tested experimentally, and the calculation efficiency and accuracy of FEM, image method and PGD algorithm are compared. The results show that, compared with the FEM and the image method, the calculation speed of the model established in this paper is faster, and it can accurately and effectively calculate the loss of rectangular Litz wire high-frequency transformers

Index Terms—Proper generalized decomposition (PGD), Complex permeability, model order reduction (MOR), Litz wire.

I. INTRODUCTION

With the rapid development of smart grid, energy internet and other future power grid technologies, the application of high-frequency transformers is more and more extensive. However, with the increase of the operating frequency, the winding loss increases significantly, which leads to high local temperature rise and low energy transmission efficiency. In order to reduce the winding loss under high-frequency magnetic field, windings with good high-frequency performance, such as Litz wire, are generally used. Therefore, the research on winding loss calculation of Litz wire at high frequency has important engineering significance and application prospect for structural optimization and heat dissipation analysis and design of transformers.

At present, the calculation methods of high frequency loss of Litz windings mainly include numerical method, analytical method and semi-analytical method. Although the numerical method is more accurate, it is time-consuming to calculate the eddy current field of Litz wire winding. The analytical method usually uses the image method to calculate the magnetic field, but the image method needs multi-layer image to calculate the inner window, which will reduce the calculation speed. While the semi-analytical method also includes numerical calculation. In order to improve the calculation efficiency, model order reduction (MOR) algorithms have been developed and used. The Proper Generalized Decomposition (PGD) algorithm, as a new generation of dimension reduction and order reduction strategy, has been developed and used in many fields. In electromagnetic computation field, the current distribution in motor slots and the inductance of rectangular conductors have been analyzed by PGD algorithm^[2]. However, there is no relevant research on the calculation of Litz wire loss of high frequency transformer.

In order to accurately and rapidly calculate the winding loss of Litz wire of the high frequency transformer, a calculation

model based on PGD algorithm and equivalent complex permeability is established in this paper. Two-dimensional models for inner and outer windows are built respectively and the different boundary conditions are considered respectively. The magnetic field calculation of 2D models are separated and reduced into two one-dimensional finite element analysis using the PGD algorithm, and the proximity effect loss of the conductor is calculated by the complex permeability. Two prototypes of different models are made to verify the accuracy of the algorithm.

II. METHOD DESCRIPTION

A. PGD space separation strategy of double 2D model

The calculation of transformer winding losses is closely related to the accurate calculation of winding magnetic fields. Considering the influence of the upper and lower yokes of the iron core on the magnetic field at the end of the winding, this paper divides the winding into two 2D models, the inner window (IW) and the outer window (OW), as shown in Fig.1. Different boundary conditions are given, and the PGD algorithm is used to solve the magnetic fields in the IW and OW regions respectively.

For the outer window of the high-frequency transformer, consider the boundary conditions,

$$\begin{cases} u_{(x=L,y)} = 0, & u_{(x,y=0)} = 0 \\ u_{(x,y=H)} = 0, & \left. \frac{\partial u}{\partial x} \right|_{x=0,y} = 0 \end{cases} \quad (1)$$

L and H are the length and width of the window solution domain, respectively. For all suitable test functions $\mathbf{A}^*$, the weighted residual form,

$$\begin{aligned} & \int_{\Omega_x \times \Omega_y} \left(\frac{\partial \mathbf{A}^*}{\partial x} \cdot \frac{\partial \mathbf{A}}{\partial x} + \frac{\partial \mathbf{A}^*}{\partial y} \cdot \frac{\partial \mathbf{A}}{\partial y} \right) dx \cdot dy \\ & = - \int_{\Omega_x \times \Omega_y} \mathbf{A}^* \cdot \mu_0 f dx \cdot dy + \int_{\Omega_y} \mathbf{A}^*(x=0,y) \cdot 0 dy \end{aligned} \quad (2)$$

For the inner window of a high-frequency transformer, consider the boundary conditions,

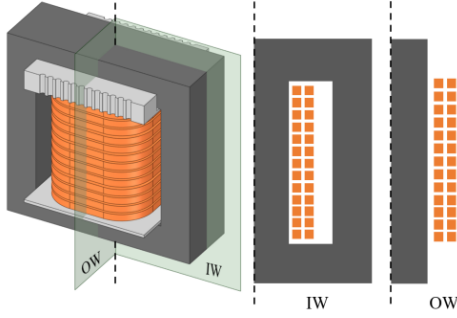


Fig.1 Cross section of inside and outside core window

$$\begin{cases} \frac{\partial u}{\partial x}\bigg|_{x=0,y} = 0, \quad \frac{\partial u}{\partial y}\bigg|_{x,y=0} = 0 \\ \frac{\partial u}{\partial x}\bigg|_{x=L,y} = 0, \quad \frac{\partial u}{\partial y}\bigg|_{x,y=H} = 0 \end{cases} \quad (3)$$

For all suitable test functions $\mathbf{A}^*$, the weighted residual form,

$$\begin{aligned} \int_{\Omega_x \times \Omega_y} \left(\frac{\partial \mathbf{A}^*}{\partial x} \cdot \frac{\partial \mathbf{A}}{\partial x} + \frac{\partial \mathbf{A}^*}{\partial y} \cdot \frac{\partial \mathbf{A}}{\partial y} \right) dx \cdot dy = - \int_{\Omega_x \times \Omega_y} \mathbf{A}^* \cdot \mu_0 f dx \cdot dy \\ + \int_{\Omega_y} \mathbf{A}^* (x=0, y) \cdot 0 dy + \int_{\Omega_y} \mathbf{A}^* (x=L, y) \cdot 0 dy \\ + \int_{\Omega_x} \mathbf{A}^* (x, y=0) \cdot 0 dy + \int_{\Omega_x} \mathbf{A}^* (x, y=H) \cdot 0 dy \end{aligned} \quad (4)$$

PGD algorithm separates the space and constructs the approximate solution of the separation form as follows,

$$\mathbf{A}^n(x, y) = \sum_{j=1}^{n-1} \mathbf{X}_j(x) \mathbf{Y}_j(y) + \mathbf{X}_n(x) \mathbf{Y}_n(y) \quad (5)$$

The test function as,

$$\mathbf{A}^*(x, y) = \mathbf{X}_n(x) \mathbf{Y}_n(y) \quad (6)$$

Take (5) (6) into (2) and (4), $\mathbf{X}(x)$ or $\mathbf{Y}(y)$ can be computed by alternating iterations until the convergence criterion $\delta(n) < \epsilon$ is satisfied.

$$\delta(n) = \frac{\| \mathbf{X}_n^t(x) \cdot \mathbf{Y}_n^t(y) - \mathbf{X}_n^{t-1}(x) \cdot \mathbf{Y}_n^{t-1}(y) \|}{\| \mathbf{X}_n^{t-1}(x) \cdot \mathbf{Y}_n^{t-1}(y) \|} < \epsilon \quad (7)$$

The approximate solution $\mathbf{A}(x, y)$ can be obtained by $\mathbf{X}(x)$ and $\mathbf{Y}(y)$, and the error operator $\varepsilon(n)$ is judged to be less than the specified tolerance $\tilde{\varepsilon}$, and the iteration stops when $\varepsilon(n) < \tilde{\varepsilon}$.

$$\varepsilon(n) = \frac{\| \mathbf{X}_n(x) \cdot \mathbf{Y}_n(y) \|}{\| \mathbf{X}_1(x) \cdot \mathbf{Y}_1(y) \|} \quad (8)$$

The vector magnetic potential $\mathbf{A}$ is calculated, the magnetic field strength $\mathbf{H}$ can be obtained.

B. High frequency Litz wire loss calculation

The skin effect loss per unit length of Litz wire can be expressed as the superposition of the loss caused by n strands of wire.

$$P_{skin} = nR_s \frac{zJ_0(z)}{4J_1(z)} \left(\frac{I_w}{n} \right)^2 = \frac{\rho}{n\pi r_s^2} \cdot \frac{zJ_0(z)}{4J_1(z)} I_w^2, \quad z = r_s \frac{(1-j)}{\delta} \quad (9)$$

where J_0 and J_1 are Bessel functions of order 0 and 1 respectively. r_s is the radius of strand; j is an imaginary unit; δ is skin depth; R_s is the DC resistance per unit length of strand. I_w is the RMS value of the alternating current.

The magnetic field in winding region is calculated by PGD algorithm, and the complex permeability was used to characterize the proximity effect of the conductor^[3]. The relationship between the imaginary part of the complex permeability and the

loss of the proximity effect in the non-permeable cross section was

$$P_{prox} = \pi f \mu'' \left(l_{IW} \int_{\Omega_c} |H|^2 dV + l_{OW} \int_{\Omega_c} |H|^2 dV \right) \quad (10)$$

where l_{IW} , l_{OW} are the equivalent length of the inner/outer core window respectively. f is the operating frequency; μ'' is the imaginary part of the complex permeability.

The complex permeability of the Litz wire can be calculated by introducing the Ollendorff formula,

$$\mu = \mu_0 \left[1 + \frac{2\eta(\dot{\mu}_r - 1)}{2 + (1-\eta)(\dot{\mu}_r - 1)} \right] \quad (11)$$

where μ_0 is the vacuum permeability; η is the filling coefficient of a single Litz wire.

$\dot{\mu}$ is the complex permeability of an isolated circular conductor,

$$\dot{\mu}_r = \mu_r \frac{J_1(z)}{zJ_1'(z)} \quad (12)$$

where μ_r is the relative permeability of the copper wire; J_1' is the derivative of a first-order Bessel function.

Finally, the total loss of the Litz wire of the high-frequency transformer is the sum of the skin effect loss and the proximity effect loss.

$$P = P_{skin} + P_{prox} \quad (13)$$

III. APPLICATION AND ANALYSIS

In order to verify the accuracy and calculation speed of the model presented in this paper, two kinds of rectangular Litz wire are used to make high-frequency transformer models. The resistances of the winding under different operating frequency are measured, and the Ohm loss can be obtained. The transformer models have been analyzed by FEM, image method and PGD algorithm respectively. The results are shown in Table 1.

TABLE 1
SOLUTION TIME OF DIFFERENT CALCULATION MODELS

Type	Time(s)		
	FEM	Image method	PGD
IW	59	30.503	0.097
OW	59	2.445	0.102

IV. CONCLUSIONS

In this paper, a high frequency transformer loss calculation model based on PGD and equivalent complex permeability is proposed, and the accuracy of the method is verified by experiments. The model can be further applied to the fast calculation required for the optimal design of high-frequency transformer structure.

References

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