

Research on Preconditioned BiCGSTAB Based on PGD and the Application in Electromagnetic Field Analysis of Three-Phase Motors

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The Proper Generalized Decomposition (PGD) algorithm combined with Jacobi preconditioned biconjugate gradient stabilized method (BiCGSTAB) is presented to analyze the transient electromagnetic field of a three-phase induction motor. In Poisson equation of transient eddy current magnetic field, the motion of motor rotor is considered using Eulerian description. The PGD algorithm that separate space domain and time domain to solve the transient eddy current magnetic field is derived, and the dimension reduction calculation of space-time separation is realized. In order to ensure the convergence of spatial domain solution, the Jacobi preconditioned BiCGSTAB is introduced to solve the asymmetric positive definite equations obtained by spatial discretization. Apply the algorithm to analyze the magnetic field of three-phase induction motor in TEAM30 problem, the effectiveness and accuracy are validated. Compared with the traditional time incremental finite element method, the computational speed and efficiency are obviously improved under the same accuracy. It is suitable for large-scale transient electromagnetic field numerical computations.

Index Terms—Proper generalized decomposition, biconjugate gradient stabilized, FEM, model order reduction.

I. INTRODUCTION

In the analysis of transient electromagnetic field for electrical equipment with large physical size, the traditional finite element method needs to discretize the space and time domain at the same time. In order to ensure the calculation accuracy, it is necessary to divide the grid of the space domain reasonably and divide the step size of the time domain appropriately, which inevitably leads to a significant increase in computation time and cost. As for the equipment with moving parts, such as motors, the involvement of rotor motion further increases the complexity and time of calculation. In order to reduce the calculation scale, model order reduction (MOR) algorithms have been developed and used, the Proper Generalized Decomposition (PGD) algorithm is a reduced-order algorithm that separates variables and solves them.

PGD solve the high-dimensional problems by decomposing them into a series of low-dimensional problems^[1], which can greatly reducing the computation and storage requirements. At present, it has been applied to the fields of fluid mechanics, structure mechanics, biomedicine and electromagnetism, etc. In the electromagnetic field analysis of motor, literature [2] considers the rotor motion by rotor angle, which is taken as an unknown variable and decomposed and calculated together with motor space structure by PGD algorithm. The interpolation method has been used to calculate the geometric changes related to the rotor motion region.

In this paper, a PGD time-space separation reduced-order model for calculating the transient electromagnetic field of three-phase asynchronous motor is proposed, and the eddy current field governing equation considering rotor motion is obtained using Euler description. The mesh changes caused by rotor motion do not require additional processing. In order to solve the asymmetric positive definite problem of space matrix caused by moving terms, Jacobi preconditioned BiCGSTAB method is introduced to improve the calculation efficiency. The model is applied to calculate the magnetic field and torque of

the motor in Team30 problem, and the results verify the correctness of the model.

II. METHOD DESCRIPTION

A. Transient electromagnetic field equation of motor considering rotor motion

In the finite element calculation of electromagnetic field, the methods to deal with the motion problem are divided into Eulerian description and Lagrangian description. For the Eulerian method, a velocity-related convection term is added to the field governing equation to consider the motion, and the computational grid remains unchanged during the motion process without any treatment. The 2D transient eddy current field equation of the motor is

$$\sigma \frac{\partial \mathbf{A}}{\partial t} + \sigma \vec{v} \nabla \mathbf{A} - \frac{\partial}{\partial x} \left(\frac{1}{\mu} \frac{\partial \mathbf{A}}{\partial x} \right) - \frac{\partial}{\partial y} \left(\frac{1}{\mu} \frac{\partial \mathbf{A}}{\partial y} \right) = \mathbf{J}_s(x, y, t) \quad (1)$$

where $\mathbf{A}$ is the magnetic vector potential; σ is the conductivity; $\vec{v}$ is the speed of the rotor; μ is for permeability; $\mathbf{J}_s(x, y, t)$ is the source current density varying with time. Finite element method is used to obtain discrete matrix equations

$$(\mathbf{K} + \mathbf{K}_p) \mathbf{A}(t) + \mathbf{M} \frac{d\mathbf{A}(t)}{dt} = \mathbf{F}(t) \quad (2)$$

where $\mathbf{K}$ is the stiffness matrix, $\mathbf{K}_p$ is the matrix considering rotor motion, $\mathbf{M}$ is the mass matrix, by solving (2), the vector magnetic potentials of all nodes can be obtained.

B. PGD algorithm for time-space dimension reduction separation

The unknown variables $\mathbf{A}$ and source current $\mathbf{J}$ in (1) both vary with space and time. The PGD algorithm constructs an approximate solution in the form of time and space separation

$$\mathbf{A}(x, y, t) = \sum_{i=1}^{n-1} \mathbf{X}_i(x, y) \cdot \mathbf{T}_i(t) + \mathbf{X}_n(x, y) \cdot \mathbf{T}_n(t) \quad (3)$$

where n is the number of sums of functional modes.

Using the Galerkin weighted residual form for (1), the test function $\mathbf{A}'$ as:

$$\mathbf{A}'(x, y, t) = \mathbf{X}_n(x, y) \cdot \mathbf{T}_n(t) \quad (4)$$

the following formula can be obtained

$$\begin{aligned}
& \int_D \frac{1}{\mu} \nabla \times \mathbf{X}_n(x, y) \cdot \nabla \times \mathbf{X}_n(x, y) dD \int_0^T T_n(t) \cdot T_n(t) dt \\
& + \int_D \sigma \mathbf{X}_n(x, y) \cdot \mathbf{X}_n(x, y) dD \int_0^T \partial_t T_n(t) \cdot T_n(t) dt \\
& + \int_D \sigma \nabla \nabla \times \mathbf{X}_n(x, y) \cdot \mathbf{X}_n(x, y) dD \int_0^T T_n(t) \cdot T_n(t) dt \\
& = \mathbf{F} - \mathbf{F}_1 - \mathbf{F}_2 - \mathbf{F}_3
\end{aligned} \quad (5)$$

where $\mathbf{F}$ is the excitation source term, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are respectively

$$\begin{aligned}
\mathbf{F}_1 &= \sum_{i=1}^{n-1} \int_D \frac{1}{\mu} \nabla \times \mathbf{X}_i(x, y) \cdot \nabla \times \mathbf{X}_n(x, y) dD \int_0^T T_i(t) \cdot T_n(t) dt \\
\mathbf{F}_2 &= \sum_{i=1}^{n-1} \int_D \sigma \mathbf{X}_i(x, y) \cdot \mathbf{X}_n(x, y) dD \int_0^T \partial_t T_i(t) \cdot T_n(t) dt. \\
\mathbf{F}_3 &= \sum_{i=1}^{n-1} \int_D \sigma \nabla \nabla \times \mathbf{X}_i(x, y) \cdot \mathbf{X}_n(x, y) dD \int_0^T T_i(t) \cdot T_n(t) dt
\end{aligned} \quad (6)$$

All functions of T are known and can be calculated numerically using trapezoidal rule as

$$\begin{cases} \int_0^1 T_n(t) \cdot T_n(t) dt = \alpha^x, \int_0^T \partial_t T_n(t) \cdot T_n(t) dt = \beta^x \\ \int_0^T i_i(t) \cdot T_n(t) dt = \xi_i^x, \int_0^T T_i(t) \cdot T_n(t) dt = \gamma_i^x \\ \int_0^T \partial_t T_i(t) \cdot T_n(t) dt = \delta_i^x \end{cases} \quad (7)$$

PGD uses alternating iterations to solve $\mathbf{X}$ and T separately. The discrete formula for solving $\mathbf{X}$ is

$$\begin{aligned}
& (\mathbf{K} \cdot \alpha^x + \mathbf{M} \cdot \beta^x + \mathbf{K}_p \cdot \alpha^x) \mathbf{X}_n \\
& = \sum_{i=1}^3 \mathbf{F}_{mi} \cdot \xi_i^x - \sum_{i=1}^{n-1} \mathbf{K} \mathbf{X}_i \cdot \gamma_i^x - \sum_{i=1}^{n-1} \mathbf{M} \mathbf{X}_i \cdot \delta_i^x - \sum_{i=1}^{n-1} \mathbf{K}_p \mathbf{X}_i \cdot \gamma_i^x
\end{aligned} \quad (8)$$

Based on $\mathbf{X}$, the discrete formula for solving T is

$$\begin{aligned}
& (\mathbf{K} \mathbf{X}_n + \mathbf{K}_p \mathbf{X}_n) T_n(t) + \mathbf{M} \mathbf{X}_n \frac{dT_n(t)}{dt} \\
& = \sum_{i=1}^3 \mathbf{F}_{mi} i_i(t) - \sum_{i=1}^{n-1} T_i(t) \mathbf{K} \mathbf{X}_i - \sum_{i=1}^{n-1} T_i(t) \mathbf{K}_p \mathbf{X}_i - \sum_{i=1}^{n-1} \frac{dT_i(t)}{dt} \mathbf{M} \mathbf{X}_i
\end{aligned} \quad (9)$$

Set an initial value $T_n^0(t)$ to begin the iterative process, when the iteration converges, $\mathbf{X}_n^p(x, y)$ and $T_n^p(t)$ can be obtained.

C. Space-time separation of motor excitation source

The excitation sources of the magnetic field in a three-phase motor are the sinusoidal currents in the spatially distributed three-phase windings, $i_k(t) = \sin(2\pi ft + (k-1)2\pi/3)$, $k=1,2,3$, so $\mathbf{F}$ is also a function of time and space.

$\mathbf{F} = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \sum_{k=1}^3 \mathbf{F}_k$, Express $\mathbf{F}_k$ as the product of space

function and space function, $\mathbf{F}_k = \mathbf{F}_{Xk} \cdot \mathbf{F}_{Tk}$.

$$\mathbf{F}_{Xk} = \int_D \mathbf{J}_{mk} \cdot \mathbf{X}_n(x, y) dD \quad (10)$$

$$\mathbf{F}_{Tk} = \int_0^T i_k(t) \cdot T_n(t) dt \quad (11)$$

$\mathbf{J}_{mk}$ is the peak current density.

D. Jacobi Preconditioned BiCGSTAB Method for PGD spatial domain solution

Due to the involvement of the moving term, the symmetry and positive definiteness of the coefficient matrix of (8) is destroyed, which makes the conjugate gradient method and Gauss-Seidel iteration no longer applicable. In this paper, the

double conjugate gradient stability method BICGSTAB is used for iterative solution, and the Jacobi preprocessing technology is used to improve the convergence speed.

The preconditioned matrix $\mathbf{M}_J$ is defined by $\mathbf{M}_J = \mathbf{D}^{-1}$, $\mathbf{D} = \text{diag}(\mathbf{A})$. Then the preconditioner BICGSTAB algorithm can be expressed as follows:

1) Calculate the initial residual

$$\mathbf{r}_0 = \mathbf{b} - \mathbf{A} \mathbf{x}_0 \quad (12)$$

2) Set initial value

$$\mathbf{r}_0^* = \mathbf{r}_0, \rho_{-1} = \alpha_{-1} = \omega_{-1} = 1, \mathbf{v}_{-1} = \mathbf{p}_{-1} = 0 \quad (13)$$

3) Calculate ρ_i , β_{i-1} and $\mathbf{p}_i$

$$\rho_i = (\mathbf{r}_i, \mathbf{r}_0^*), \beta_{i-1} = \left(\frac{\rho_i}{\rho_{i-1}} \right) \left(\frac{\alpha_{i-1}}{\omega_{i-1}} \right) \quad (14)$$

$$\mathbf{p}_i = \mathbf{r}_i + \beta_{i-1}(\mathbf{p}_{i-1} - \omega_{i-1}\mathbf{v}_{i-1}) \quad (15)$$

4) obtain $\hat{\mathbf{p}}$ by solving

$$\mathbf{M}_J \hat{\mathbf{p}} = \mathbf{p}_i \quad (16)$$

5) Calculate $\mathbf{v}_i$, α_i and $\mathbf{s}_i$

$$\mathbf{v}_i = \mathbf{A} \hat{\mathbf{p}}, \alpha_i = \rho_i / (\mathbf{v}_i, \mathbf{r}_0^*), \mathbf{s}_i = \mathbf{r}_i - \alpha_i \mathbf{v}_i \quad (17)$$

6) obtain $\mathbf{z}_i$ by solving

$$\mathbf{M}_J \mathbf{z}_i = \mathbf{s}_i \quad (18)$$

7) Calculate $\mathbf{t}_i$ and ω_i

$$\mathbf{t}_i = \mathbf{A} \mathbf{z}_i, \omega_i = (\mathbf{t}_i, \mathbf{s}_i) / (\mathbf{t}_i, \mathbf{t}_i) \quad (19)$$

8) Calculate $\mathbf{x}_{i+1}$

$$\mathbf{x}_{i+1} = \mathbf{x}_i + \alpha_i \hat{\mathbf{p}} + \omega_i \mathbf{z}_i \quad (20)$$

9) If $(\mathbf{x}_{i+1})$ is accurate enough) stop iteration, Else

$$\mathbf{r}_{i+1} = \mathbf{s}_i - \omega_i \mathbf{t}_i \quad (21)$$

go to 3)

End do.

III. APPLICATION AND ANALYSIS

The model and program have been applied to calculate the magnetic field and torque of induction motor in TEAM 30 problem. The magnetic field distribution and rotor torque are shown in Fig.1-2.

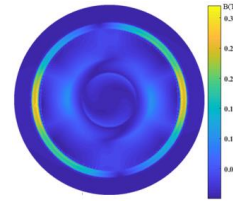


Fig. 1. The main magnetic field of transformer

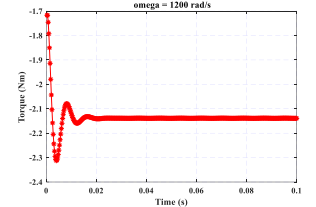


Fig. 2 Rotor torque

IV. CONCLUSIONS

The transient electromagnetic field calculation model of PGD three-phase asynchronous motor with spatiotemporal separation is established in this paper, and the magnetic field and torque of the motor are quickly calculated.

References

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