

Finite Element Analysis of Metasurface-Coupled MCR-WPT System with Generalized Sheet Transition Conditions

X. Xu¹ and S. Yang^{2*}

¹School of Electrical Engineering, China University of Mining and Technology, Xuzhou, 221116 China, xwxu@cumt.edu.cn

²College of Electrical Engineering, Zhejiang University, Hangzhou, 310027 China, eesyyang@zju.edu.cn

A frequency-domain finite element method incorporating Generalized Sheet Transition Conditions (GSTCs) is proposed, which is rather simple and particular well-suited for the multiscale simulation of magnetically coupling resonant wireless power transfer (MCR-WPT) system. The discontinuities in electromagnetic fields across a metasurface as represented by the GSTCs are modeled by assigning nodes to one side of the metasurface. The FEM-GSTCs formulation is derived and implemented. For simplicity, the proposed formulation is presented for analysis of 2-D problems consisting of monoisotropic metasurfaces. The extension of this method 3-D problems or bianisotropic metasurfaces is straightforward.

Index Terms—Metasurfaces, Finite element analysis, Frequency-domain analysis, Wireless power transmission.

I. INTRODUCTION

METASURFACES have been the popular research topic in wireless power transmission and millimeter-wave fields during the past twenty years^[1]. However, the simulation of metasurfaces is a typical multiscale problem, as the structure of metasurfaces is usually much smaller than the wavelength, while the size of wireless energy transmission systems is usually comparable to the wavelength. Therefore, accurate modeling of metasurface-coupled wireless energy transmission systems requires huge computational resources. Generalized Sheet Transition Conditions (GSTCs) is a widely used homogenization technique that simplifies the modeling of metasurfaces with complex^[2]. Some previous researches have applied the GSTCs to modeling metasurfaces in electromagnetic simulation in recent years. The spectral domain integral equation (IE) with GSTCs and FDTD method with GSTCs is proposed^{[3]-[5]}. However, metasurfaces considered in these numerical methods are restricted to simple geometry. Therefore, a FEM-GSTCs is proposed in this paper for the multiscale simulation of magnetically coupling resonant wireless power transfer (MCR-WPT) system.

II. FREQUENCY-DOMAIN FINITE ELEMENT METHOD WITH GSTCS

A. Susceptibility Modelling of Metasurface

The frequency-domain GSTCs for a metasurface models the effects of the metasurface scattering particles in terms of effective polarizations. The metasurface is assumed to be a zero-thickness sheet producing a discontinuity in the tangential components of the E and H fields across the surface. For a metasurface placed perpendicular to the z-direction of a cylindrical coordinate system, The frequency domain GSTCs excluding the excitation of normal polarizations are given by^[6]

$$\hat{\mathbf{z}} \times \Delta \mathbf{H} = j\omega \mathbf{P}_{\parallel} - \hat{\mathbf{z}} \times \nabla_{\parallel} \mathbf{M}_z \quad (1)$$

$$\hat{\mathbf{z}} \times \Delta \mathbf{E} = -j\omega \mu_0 \mathbf{M}_{\parallel} - \hat{\mathbf{z}} \times \nabla_{\parallel} (\mathbf{P}_z / \varepsilon_0) \quad (2)$$

Where $\nabla_{\parallel} = \hat{x} \partial / \partial x + \hat{y} \partial / \partial y$. $\mathbf{P}$ and $\mathbf{M}$ are the electric and magnetic polarization densities, respectively, and is expressed as

$$\mathbf{P} = \varepsilon_0 \overline{\chi}_{ee} \cdot \mathbf{E}_{av} + (1/c_0) \overline{\chi}_{em} \cdot \mathbf{H}_{av} \quad (3)$$

$$\mathbf{M} = \overline{\chi}_{mm} \cdot \mathbf{H}_{av} + (1/\eta_0) \overline{\chi}_{me} \cdot \mathbf{E}_{av} \quad (4)$$

Where $\mathbf{P}$ (C/m²) is the material polarizations via the electric polarization density vector, $\mathbf{M}$ (A/m) is the magnetic polarization density vector, ε_0 and η_0 are the vacuum permittivity and impedance, respectively. c_0 is the speed of light in a vacuum, and $\overline{\chi}_{ee}$, $\overline{\chi}_{mm}$, $\overline{\chi}_{em}$ and $\overline{\chi}_{me}$ are the electric, magnetic, magnetic-to-electric, and electric-to-magnetic susceptibility tensors, respectively. The average fields $\mathbf{H}_{av}$ and $\mathbf{E}_{av}$ are the arithmetic average of the fields on the two sides of the metasurface,

$$\mathbf{H}_{u,av} = \mathbf{H}_u^i + \mathbf{H}_u^r + \mathbf{H}_u^t / 2, \quad u = x, y, z \quad (5)$$

$$\mathbf{E}_{u,av} = \mathbf{E}_u^i + \mathbf{E}_u^r + \mathbf{E}_u^t / 2, \quad u = x, y, z \quad (6)$$

where the superscripts i , r , and t denote incident, reflected and transmitted fields.

For a large number of practical metasurfaces, $\mathbf{P}_z = \mathbf{M}_z = 0$, and hence the susceptibility tensors reduce to 2×2 transverse tensors, substitution of (3) and (4) into (1) and (2) results in bianisotropic GSTC synthesis equations in the frequency-domain as

$$\begin{pmatrix} -\Delta H_y \\ \Delta H_x \end{pmatrix} = j\omega \varepsilon_0 \overline{\chi}_{ee} \begin{pmatrix} E_{x,av} \\ E_{y,av} \end{pmatrix} + j\omega \sqrt{\varepsilon_0 \mu_0} \overline{\chi}_{em} \begin{pmatrix} H_{x,av} \\ H_{y,av} \end{pmatrix} \quad (7)$$

$$\begin{pmatrix} \Delta E_y \\ -\Delta E_x \end{pmatrix} = j\omega \mu_0 \overline{\chi}_{mm} \begin{pmatrix} H_{x,av} \\ H_{y,av} \end{pmatrix} + j\omega \sqrt{\varepsilon_0 \mu_0} \overline{\chi}_{me} \begin{pmatrix} E_{x,av} \\ E_{y,av} \end{pmatrix} \quad (8)$$

In this work, we consider the homoanisotropic metasurface, which possesses only diagonal nonzero susceptibility tensors. Therefore, the homoanisotropic GSTCs are obtained as

$$\Delta H_y = -j\omega \varepsilon_0 \overline{\chi}_{ee}^{xx} E_{x,av} \quad (9)$$

$$\Delta H_x = j\omega \varepsilon_0 \overline{\chi}_{ee}^{yy} E_{y,av} \quad (10)$$

$$\Delta E_x = -j\omega \mu_0 \overline{\chi}_{mm}^{yy} H_{y,av} \quad (11)$$

$$\Delta E_y = j\omega\mu_0\chi_{mm}^{xx}H_{x,av} \quad (12)$$

Where $\Delta H_y = H_{y,t} - (H_{y,i} + H_{y,r})$, and so on. A metasurface with the susceptibilities in (9)-(12) will exactly produce the specified reflected and transmitted transverse components of the fields when the metasurface is illuminated by the specified incident field.

B. Formulation of GSTC-Finite Element Method

As a time-harmonic electromagnetic field under sinusoidal steady state, the governing equations for a magnetoquasistatic MCR-WPT system are given by Maxwell's equations:

$$\nabla \times \dot{\mathbf{H}} = \sigma \dot{\mathbf{E}} + \dot{\mathbf{J}}_s, \quad (13)$$

$$\nabla \times \dot{\mathbf{E}} = -j\omega\mu\dot{\mathbf{H}}, \quad (14)$$

$$\nabla \cdot \dot{\mathbf{B}} = 0. \quad (15)$$

Alongside these equations, we also consider the constitutive relations,

$$\dot{\mathbf{B}} = \mu\dot{\mathbf{H}}. \quad (16)$$

Applying Galerkin's method to (13)-(15) results in a system of equations for nodal values of H_y , which can be written as

$$\mathbf{K}_1 \dot{\mathbf{H}} = \mathbf{P}_1 \dot{\mathbf{E}} + \dot{\mathbf{F}}, \quad (17)$$

$$\mathbf{K}_2 \dot{\mathbf{E}} = \mathbf{P}_2 \dot{\mathbf{H}}, \quad (18)$$

$$\mathbf{K}_3 \dot{\mathbf{H}} = 0. \quad (19)$$

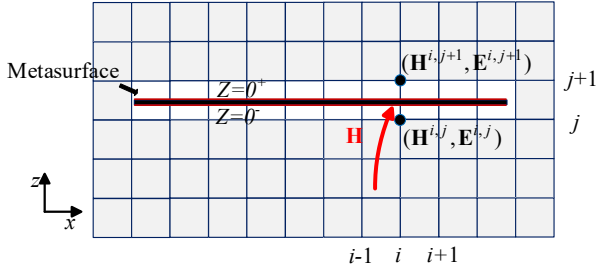


Fig.1. FEM 2D computational domain with a metasurface

For demonstration, consider the case of a 2-D computational domain with a metasurface lying in-between two rows of nodes, as depicted in Fig. 1. In order to satisfy the condition of zero thickness, it is necessary to incorporate the metasurface as a virtual entity within the grid by placing it in between field nodes. The presence of the metasurface is then considered by interconnecting the nodes before and after the metasurface via the GSTCs. Expanding the expressions of the average and difference of the fields in (9)-(12), we get

$$H_y^{i,j+1} - H_y^{i,j} = -j\omega\epsilon_0\chi_{ee}^{xx}(E_x^{i,j+1} + E_x^{i,j})/2, \quad (20)$$

$$H_x^{i,j+1} - H_x^{i,j} = j\omega\epsilon_0\chi_{ee}^{yy}(E_y^{i,j+1} + E_y^{i,j})/2, \quad (21)$$

$$E_x^{i,j+1} - E_x^{i,j} = -j\omega\epsilon_0\chi_{mm}^{yy}(H_y^{i,j+1} + H_y^{i,j})/2, \quad (22)$$

$$E_y^{i,j+1} - E_y^{i,j} = j\omega\epsilon_0\chi_{mm}^{zz}(H_x^{i,j+1} + H_x^{i,j})/2. \quad (23)$$

The discontinuity induced by the metasurface is then incorporate by defining a virtual node at $z = 0+$ such that $H_x^{i,0+}$ and $H_y^{i,0+}$ are used instead of $H_x^{i,j}$ and $H_y^{i,j}$ in (13), also $E_x^{i,0+}$ and $E_y^{i,0+}$ are used instead of $E_x^{i,j}$, and $E_y^{i,j}$ in (14).

III. NUMERICAL EXAMPLE

To facilitate the verification of the proposed method, we consider a 2-D axisymmetric computational problem as shown in Fig. 2. The distribution of magnetic field is observed for the case where the metasurface is at the center between the transmission and receiver coils (0.15m from the coils).

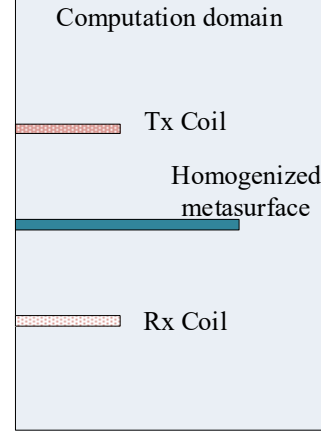


Fig.2. A Metasurface-coupled MCR-WPT system

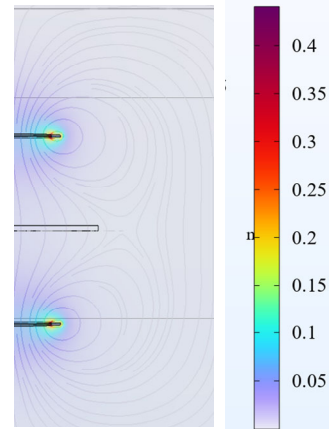


Fig.3. Magnetic flux density of the MCR-WPT system

REFERENCES

- [1] H. T. Chen, A. J. Taylor, and N. Yu, "A review of metasurfaces: physics and applications," *Rep. Prog. Phys.*, vol. 79, no. 7, pp. 1-40, Jul. 2016.
- [2] J. Maxwell, *A Treatise on Electricity and Magnetism*. Cambridge, U.K.: Cambridge Univ. Press, 2010.
- [3] J. Chen, L. E. Tobon, M. Chai, J. A. Mix, and Q. H. Liu, "Efficient implicit-explicit time stepping scheme with domain decomposition for multiscale modeling of layered structures," *IEEE Trans. Compon., Packag., Manuf. Technol.*, vol. 1, no. 9, pp. 1438-1446, Sep. 2011.
- [4] J. Wang and Q. Ren, "A 3-D hybrid Maxwell's equations finite difference time domain (ME-FDTD)/wave equation finite-element time domain (WE-FETD) method," *IEEE Trans. Antennas Propag.*, vol. 71, no. 6, pp. 5212-5220, Jun. 2023.
- [5] J. Mi, Q. Ren, and D. Su, "Parallel subdomain-level DGTD method with automatic load balancing scheme with tetrahedral and hexahedral elements," *IEEE Trans. Antennas Propag.*, vol. 69, no. 4, pp. 2230-2241, Apr. 2021.
- [6] K. Achouri and C. Caloz, *Electromagnetic Metasurfaces: Theory and Applications*. Hoboken, NJ, USA: Wiley, 2021.