

Numerical Techniques for Finite Element Modeling of Superconductors

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This paper deals with numerical techniques for finite element modeling of superconductors. For that, we employ different methods: full storage (FS), compressed storage (CRS and CCS) and normalized formulation (NF). After optimizing and testing the program, we observe the computation time. Then we compare the results obtained by using different methods for various important parameters: the critical current density, the maximum magnetic induction and the angle of the applied magnetic field.

***Index Terms*—Finite element analysis, Numerical simulation, Optimization methods, Superconducting filaments.**

I. INTRODUCTION

AT PRESENT, superconductors are widely used in various advanced applications due to their unique properties of zero electrical resistance and Meissner effect such as Magnetic Resonance Imaging (MRI), Superconducting Magnetic Energy Storage (SMES), Magnetic levitation (Maglev), Large Hadron Collider (LHC), International Thermonuclear Experimental Reactor (ITER), Quantum bits (Qubits) etc.

In order to study and analyze the electromagnetic behavior in superconductors, a model based on the generalization of Bean's model has been developed at LGEP (now called GeePs) in France [1]-[3]. This model has been integrated into a finite element software designed for cartesian and axisymmetric geometries. While the software is generally used to model the superconductors at low critical temperature (LTS), it can also be applied to model at high critical temperature (HTS). This software could be able to compute the electromagnetic field of a superconducting device using the finite element method (FEM) [4]. The discretization of time by Euler's scheme leads us to solve at each time step a nonlinear algebraic system. To do this, we must solve the fixed-point problem obtained for the magnetic field by an iterative method. At each iteration, we are led to solve a convex and strongly nonlinear minimization problem in the electric field by Gauss-Seidel algorithm. The computation time is generally significant, we seek to optimize the algorithm. To do this, we use condensation techniques that the mass matrix is diagonal. With this software, we are able to process different geometries for example, a superconducting filament of different forms (rectangular, circular and elliptical), a superconducting tube, a superconducting multifilamentary strand etc. For this last case, the coupling effects between filaments can be considered [5]-[8]. In this work, our goal is to reduce the computation time of our software that can solve all problems with a transverse magnetic field varied as a function of time in 2D.

In recent years, several software programs have used the finite difference method (FDM), finite element method (FEM) and finite volume method (FVM) to simulate and analyze the superconducting materials. For example, the world-famous FLUX, OPERA, ANSYS and COMSOL commercial software

have used the finite element analysis for electromagnetic simulations. In addition, MATLAB has also been used to simulate superconductors. Furthermore, we find at CERN the ROXIE program that is used to calculate electromagnetic fields. This program combines the finite element method and the boundary element method (BEM-FEM), powerful optimization algorithms, and CAD/CAM interfaces. The numerical solutions have been compared with the analytical solutions. Several analytical models exist worldwide, but the best-known models are Bean's model, Kim's model, and J^n model (the power law). A method based on Bean's model was developed by Brandt, this method is much more used worldwide, as well as Norris's method. And for the iteration method, in addition to the Gauss-Seidel method that we used, the Newton-Raphson method is much more used worldwide as well.

II. NUMERICAL MODELING

The electromagnetic behavior of superconductors can be described by Maxwell's equations relating to the electric field (E), the current density (J), the magnetic field (H), and the magnetic induction (B) [3]. It must be supplemented by the relations $B-H$ and $E-J$. For that, we use an altering Bean's model to get a well-behaved graph as in [2]. For solving the problems, we propose a computer algorithm as in Fig. 1 [4] where V and M are the potential and the magnetization respectively. For high performance computing, we developed our program for different techniques by using FORTRAN. This programming language supports scientific and engineering applications such as numerical computation and finite element analysis.

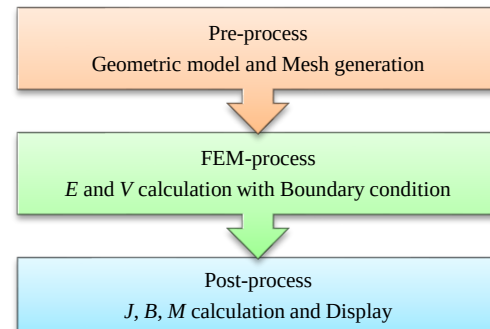


Fig. 1. Problem solving processes by using a proposed computer algorithm.

In order to reduce the computation time, we used several optimization methods. At first, we reduced the number of unknowns by eliminating those located in the air (matrix condensation). Second, we used condensation techniques so that the mass matrix is diagonal. Third, to further reduce the computation time, we changed the data storage from full storage (FS) to compressed sparse row (CSR) or compressed row storage (CRS) and compared to compressed sparse column (CSC) or compressed column storage (CCS) [9]. For FS, we store all the terms, even the zero terms, the latter causes us a lot of problems because we lose too much computation time when we do a multiplication of the matrices. On the other hand, for CRS or CCS, we only store the terms that are not zero. Then, it no longer poses a problem when we do a multiplication of the matrices. Finally, we applied the normalized formulation (NF) method to reduce even more the computation time.

III. RESULTS

After optimizing and testing our program for different techniques, we can clearly see in Fig. 2, 3 and 4 a comparison of the computation time for various parameters: the critical current density (J_c), the maximum magnetic induction (B_{max}) and the angle of the applied magnetic field (θ) between full storage, compressed storage (CRS and CCS) and normalized formulation method. When we look at the performance of the program, the program with NF, CRS and CCS could reduce the computation time by about 4 times compared to the original program with FS. This also depends on the performance of the computer used and the number of elements (mesh size).

IV. CONCLUSION

The use of superconductors shows now the real applications in several areas such as MRI machines, Maglev trains, LHC accelerators, Quantum computing etc. Then, we have to seek to model these materials in order to be able to study and analyze all the aspects of them. The optimization methods such as CRS, CCS and NF help us accelerate these studies.

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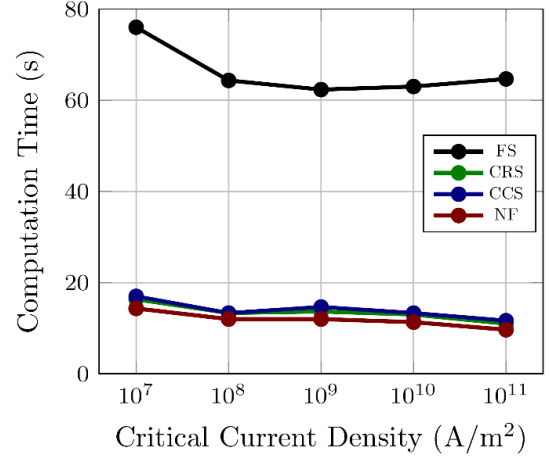


Fig. 2. Comparison of the computation time versus the critical current density between different numerical techniques (FS, CRS, CCS and NF).

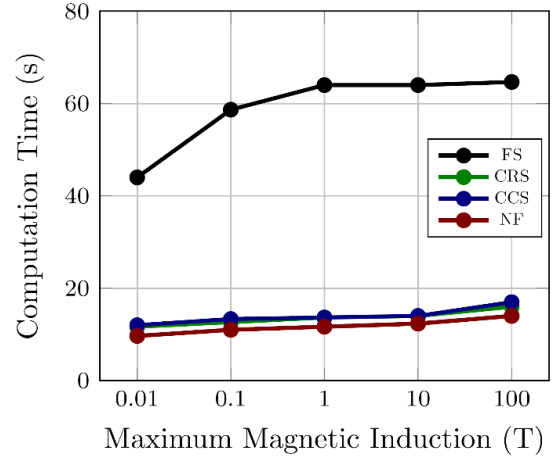


Fig. 3. Comparison of the computation time versus the maximum magnetic induction between different numerical techniques (FS, CRS, CCS and NF).

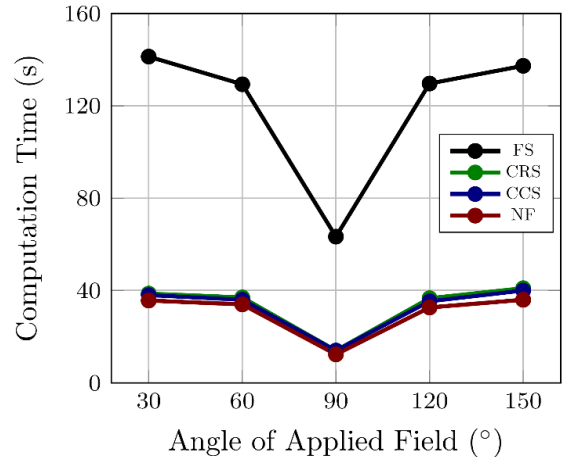


Fig. 4. Comparison of the computation time versus the angle of the applied field between different numerical techniques (FS, CRS, CCS and NF).