

Model Order Reduction of an electrical machine on a wide load range using POD combined with ECSW for fast dataset generation

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Machine learning methods are an interesting tool for automatic fault detection in electrical machines, based on the measurement of a physical quantity of interest, but need large datasets to be accurate. As there might not be available measurements for some loads or defects, numerical simulations based on the Finite Element (FE) method may be used as an alternative for dataset generation, but numerous solving of nonlinear magnetostatic problems coupled with a dynamical circuit equation system to represent the load leads to huge computational times. Then, Model Order Reduction (MOR) approaches based on the Proper Orthogonal Decomposition (POD) coupled with the interpolation method Energy Conservating Sampling with Weight (ECSW) enables to reduce the computational time. In this communication, we propose a methodology to construct a reduced model of a generator in the nonlinear case, valid on a wide range of operating points. The reduced model is evaluated in terms of accuracy on local quantities of interest and speedup, compared to a FE model.

Index Terms—Finite element modeling, Reduced order model, Electric machines, Nonlinear magnetics

I. INTRODUCTION

In a context of low carbon electricity production, rotating machines are essential components of hydraulic and nuclear power plants, and their failure often result in the complete shutdown of the energy production facility. It is therefore critical to monitor the health condition of such devices, by analyzing probe signals measuring the evolution of physical quantities over time (e.g., magnetic flux, temperature, vibrations). Then, machine learning methods can be implemented to automatically detect defaults but necessitate important datasets to be trained. Numerical methods like the Finite Element (FE) method are extensively used to analyze the electromagnetic behavior of transformers and rotating machines and can generate training data in both healthy and faulty cases [1]. Although FE models can generate valuable datasets for machine learning applications, their significant computational time is a huge drawback. To address this, model order reduction (MOR) methods have been developed, with the most common being Proper Orthogonal Decomposition (POD), based on the definition of a reduced basis. For nonlinear problems, additional methods such as the Energy Conservating Sampling with Weight (ECSW) have been developed to further reduce computational time [1]. The effectiveness of this method for industrial problems (with highly nonlinear domains, large magnetic field variations, and movement considerations) has been demonstrated in [3] to speed up the simulation of an induction machine around the nominal operating point.

In this study, we apply POD combined with ECSW to a nonlinear magnetostatic problem of a generator, coupled with the external circuit equation where the load has a wide variation. The reduced model generates precise results for a wide range of operating configurations (i.e., furniture and absorption of reactive power) and its low computational time allows for a significant speedup of signal dataset generation. In this communication, we first present the FE model of the electrical machine, followed by the methodology for constructing the

reduced model. Finally, we evaluate its performance by comparing it with the FE model.

II. MAGNETOSTATIC PROBLEM

Let consider a non-linear magnetostatic problem of an electrical machine using the vector potential formulation, and define $\mathbf{A}$ the vector potential, $\nu(\mathbf{B})$ the reluctivity depending on the magnetic flux density $\mathbf{B} = \text{curl}(\mathbf{A})$ and N_j and i_j the unit source vector and the current associated with the j^{th} winding and N_{st} the number of windings. The strong formulation to be solved is:

$$\text{curl}(\nu(\mathbf{B})\text{curl}(\mathbf{A})) = \sum_{j=1}^{N_{st}} N_j i_j, \quad (1)$$

To account for the external circuit connected at the rotor and stator windings, we add a circuit equation allowing to impose the voltage V_j at the terminals of the winding j :

$$\frac{d\Phi_j}{dt} + L \frac{di_j}{dt} + C \int i_j dt + R i_j = V_j \quad (2)$$

with R the resistance of windings, C the load capacitance, L the load inductance and Φ_j the linkage magnetic flux of the j^{th} winding depending on $\mathbf{A}$:

$$\Phi_j = \int_D \mathbf{A} \cdot \mathbf{N}_j dx. \quad (3)$$

A 2D mesh of the machine with $n_e = 232424$ elements is considered and the FE method with an implicit Euler scheme are applied to the previous governing equations (1) and (2) to obtain a non-linear system of N equations to be solved at each time step k . We denote $s_{A_k} \in \mathbb{R}^N$ the vector of the unknowns, which are the potential values at the nodes at this time step.

III. CONSTRUCTION OF THE REDUCED MODEL

A. Structure

We can derive a reduced model from the FE model using a reduced basis $\Psi \in \mathbb{R}^{N \times m}$ with m basis vectors. We introduce the approximation of the solution vector s_{A_k} in the basis Ψ by

a vector $\bar{s}_{A_k}$ such that $\bar{s}_{A_k} = \Psi s_{A_k}^r$, where $s_{A_k}^r \in \mathbb{R}^m$ is the representation of $\bar{s}_{A_k}$ in the reduced basis. Then, replacing s_{A_k} by its approximation $\bar{s}_{A_k}$ and applying the Ritz-Galerkin method on the FE system, we obtain the reduced model:

$$\mathbf{M}^{v,r} s_{A_k}^r - \sum_{j=1}^{N_{st}} F_j^r i_{j_k} = 0, \quad (4)$$

$$\begin{aligned} \frac{(F_j^r)^t}{\Delta t} s_{A_k}^r + \left[\frac{L}{\Delta t} + C \Delta t + R \right] i_{j_k} \\ = V_{j_k} + \frac{(F_j^r)^t s_{A_{k-1}}^r + L i_{j_{k-1}}}{\Delta t}. \end{aligned} \quad (5)$$

Where Δt is the time step, $\mathbf{M}^{v,r} = \Psi^t \mathbf{M}^v \Psi$ of size $m \times m$ deduced from the FE $N \times N$ matrices $\mathbf{M}^v$ accounting for the magnetic behavior and rotor movement, and $F_j^r = \Psi^t F_j \in \mathbb{R}^m$ depending on the FE source vector $F_j \in \mathbb{R}^N$. This projection of the FE equation system significantly reduces its size from N to m , thereby decreasing the computational time. However, the nonlinearity of the matrix $\mathbf{M}^v$ which depends on s_{A_k} requires multiple evaluations and projections in the FE space, resulting in a substantial computation time. To mitigate this issue, interpolation methods can be applied. We define the approximation $\bar{G}_r(\bar{s}_{A_k})$ of the reduced nonlinear term $G_r(\bar{s}_{A_k}) = \mathbf{M}^{v,r} s_{A_k}^r \in \mathbb{R}^m$ of equation (4), calculated from a small set ϵ of n_ϵ elements in the mesh such as [3]:

$$\bar{G}_r(\bar{s}_{A_k}) = \Psi^t \sum_{e \in \epsilon} \mathbf{L}_e \zeta_e \mathbf{g}_e(\bar{s}_{A_k}) \quad (6)$$

Where $\mathbf{g}_e(\cdot)$ is the contribution of the element e to the nonlinear term, ζ_e is the associated weight and $\mathbf{L}_e$ is the localization matrix. The construction of $\bar{G}_r(\bar{s}_{A_k})$ only requires the evaluation of $n_\epsilon \ll n_e$ components, significantly reducing computational cost.

B. Specificities

In the standard POD method, a reduced basis is deduced from a snapshot matrix $\mathbf{S} \in \mathbb{R}^{N \times T}$ gathering T well-chosen FE solutions s_{A_k} . Based on the Singular Value Decomposition (SVD) of the snapshot matrix $\mathbf{S} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^t$, the reduced basis Ψ is defined by the m first columns of $\mathbf{U}$. As the field variations show different dynamics in the rotor and stator depending on the movement, we define $s_{A_k}^{rot}$ the restriction of s_{A_k} to the rotor's unknowns, and $s_{A_k}^{sta}$ the restriction of s_{A_k} to the stator's unknowns, and construct the two snapshot matrices $\mathbf{S}_{rot}$ and $\mathbf{S}_{sta}$ from which we deduce the bases Ψ_{rot} of size m_{rot} and Ψ_{sta} of size m_{sta} . The final basis is then constructed as

$$\Psi = \begin{pmatrix} \Psi_{rot} & \mathbf{0} \\ \mathbf{0} & \Psi_{sta} \end{pmatrix} \quad (7)$$

and the link between the two domains is naturally performed by the Overlapping method used to account for the movement.

IV. EVALUATION OF THE REDUCED MODEL

To construct a reduced model, we perform FE simulations of the electrical machine described in Section II under no-load, short-circuit, and reactive power absorption conditions (capacitive load). These simulations generate the snapshot

matrices $\mathbf{S}_{rot}$ and $\mathbf{S}_{sta}$, containing solutions collected during the transient state. We then calculate the reduced basis Ψ , enabling the construction of a POD reduced model with a size of $m = 200$ (with $m_{rot} = 40$ and $m_{sta} = 160$), compared to the original FE system dimension of $N = 117473$. Subsequently, the set ϵ of 2645 elements and associated weights ζ are determined to construct the POD-ECSW reduced model.

To evaluate the reduced model, we compare the time evolution of local electromotive forces (e.m.f.) at 2 positions in the air gap of the machine: close to the rotor (pos. 1), and close to the stator (pos. 2), as it is monitored for equipment diagnosis by means of local flux sensors. We compute the relative error between the POD-ECSW model and the FE model, averaged over a mechanical period in steady-state for each of the 500 load points considered, and present the results on Fig. 1. Furthermore, we present the calculation time for one load point and speedup on Table 1. We can see that the proposed approach enables to reduce by almost 20 the calculation time to generate the dataset. The e.m.f. is a local value obtained by derivation of the flux, making it difficult to approximate precisely. The errors shown here are acceptable.

TABLE 1. COMPUTATION TIMES AND SPEEDUP

	time (h)	speedup
FE model	11.15	-
POD-ECSW model	0.56	19.8

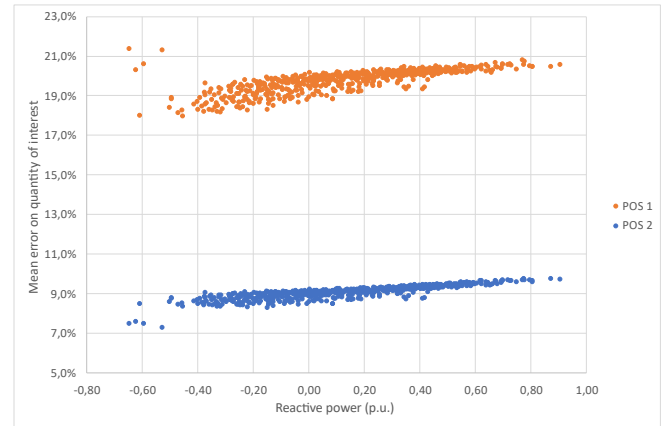


Fig. 1 - Accuracy of the reduced model for all 500 load points

CONCLUSION

We applied the POD-ECSW methodology to construct a reduced model of a generator in nonlinear case. The results show a good accuracy of the reduced model on a wide load range and a significant speedup.

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