

Analyses of Unequal Tooth-Slots PMSM Magnetomotive Force and Magnetic Flux Density and Winding Topology Optimization

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This paper derives the synthesized magnetomotive force of the permanent magnet synchronous motor with unequal tooth and slot structures under both three-phase symmetric and asymmetric currents, and provides the magnetic flux density expression when three-phase symmetric currents are applied. Finite element calculations show that applying three-phase symmetric currents generates reverse rotating magnetomotive force, and the current power angle corresponding to the maximum average output torque is related to the winding configuration. However, applying three-phase asymmetric currents can nullify the reverse rotating magnetomotive force. Using an 8-pole, 48-slot motor as an example, the effects of different winding configurations on the motor's magnetomotive force and output torque are studied, and the optimal winding configuration is presented.

Index Terms—Electromagnetic analysis, magneto motive force, magnetic flux density, permanent magnet motors, winding.

I. INTRODUCTION

RECENTLY, permanent magnet synchronous motors (PMSMs) have been used in many industrial fields [1]. Cogging torque is one of the key factors affecting motor operation, which can significantly increase torque ripple. In references [3] and [4], methods are proposed to reduce cogging torque in PMSMs by using unequal tooth widths and unequal slot widths, respectively. These methods involve making the width of one tooth (slot) different from the others while keeping the total tooth (slot) width unchanged. The novel unequal teeth and slots PMSM has the excellent performance of weakening the cogging torque, but its unequal width stator teeth and slots structure causes asymmetry in the winding structure of the motor, which leads to changes in the synthesized magnetic dynamic potential, and thus affects the magnetic flux density. Due to the presence of wide stator teeth and slots, the unequal tooth-slot PMSM has multiple armature winding configurations. Different armature winding configurations generate different synthesized magnetic motive forces. In the design of unequal tooth-slot PMSMs, it is necessary to consider the impact of different armature winding configurations on motor performance. Analytical calculations are required to determine the optimal winding configuration to improve the average output torque and reduce torque ripple.

II. ANALYSIS OF MAGNETOMOTIVE FORCE AND AIR-GAP FLUX DENSITY IN UNEQUAL TEETH AND SLOTS PMSMS

In the unequal teeth and slots PMSM, the three-phase windings are asymmetric. After three-phase symmetric currents are applied, the fundamental magnetomotive force generated by the windings has a spatial angular difference that is not 120° . Using the A-phase winding axis as the reference, virtual axis lines for the B-phase and C-phase windings are defined symmetrically with the A-phase winding axis. The angular difference between the A-phase axis and the virtual axes of the B-phase and C-phase windings is electrical 120° as shown in Fig. 1. The angle between the actual and virtual axes of the B/C phase windings is θ_B/θ_C .

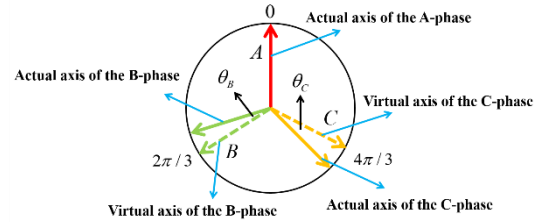


Fig. 1 The winding axis of the unequal tooth and slot PMSM.

Apply a three-phase symmetrical current to the unequal teeth and slots PMSM. Taking the origin of the spatial coordinate θ_s on the A-phase winding axis, the pulsating magnetomotive force can be decomposed as:

$$\begin{aligned} f_{A1} &= \frac{1}{2} F_{\phi 1} \cos(\omega t - \theta_s + b) + \frac{1}{2} F_{\phi 1} \cos(\omega t + \theta_s + b) \\ f_{B1} &= \frac{1}{2} F_{\phi 1} \cos(\omega t - \theta_s - \theta_B + b) + \frac{1}{2} F_{\phi 1} \cos(\omega t + \theta_s + \theta_B + b - 4\pi/3) \\ f_{C1} &= \frac{1}{2} F_{\phi 1} \cos(\omega t - \theta_s - \theta_C + b) + \frac{1}{2} F_{\phi 1} \cos(\omega t + \theta_s + \theta_C + b - 2\pi/3) \end{aligned} \quad (1)$$

The first term is the forward rotating magnetomotive force, the second term is the reverse rotating magnetomotive force. The forward rotating magnetomotive force has different spatial phases and cannot be directly added. The reverse rotating magnetomotive force does not have a 120° spatial phase difference and cannot be directly canceled out. The synthesis of the three-phase fundamental forward rotating magnetomotive force and reverse rotating magnetomotive force are:

$$\begin{aligned} f_{fp}(\theta_s, t) &= \frac{1}{2} F_{\phi 1} [\cos(-b) + \cos(\theta_B - b) + \cos(\theta_C - b)] \cos(\omega t - \theta_s) \\ &+ \frac{1}{2} F_{\phi 1} [\sin(-b) + \sin(\theta_B - b) + \sin(\theta_C - b)] \cos\left(\omega t - \theta_s - \frac{\pi}{2}\right) \end{aligned} \quad (2)$$

$$\begin{aligned} f_{fn}(\theta_s, t) &= \frac{1}{2} F_{\phi 1} \left[\begin{aligned} &\cos(\omega t + \theta_s) \cos(b) \\ &+ \cos(\omega t + \theta_s - 4\pi/3) \cos(\theta_B + b) \\ &+ \cos(\omega t + \theta_s - 2\pi/3) \cos(\theta_C + b) \end{aligned} \right] \\ &- \frac{1}{2} F_{\phi 1} \left[\begin{aligned} &\cos(\omega t + \theta_s - \pi/2) \sin(b) \\ &+ \cos(\omega t + \theta_s - 4\pi/3 - \pi/2) \sin(\theta_B + b) \\ &+ \cos(\omega t + \theta_s - 2\pi/3 - \pi/2) \sin(\theta_C + b) \end{aligned} \right] \end{aligned} \quad (3)$$

The synthesized forward rotating magnetomotive force when three-phase symmetric currents are applied to the unequal tooth and slot PMSM is shown in Fig. 2.

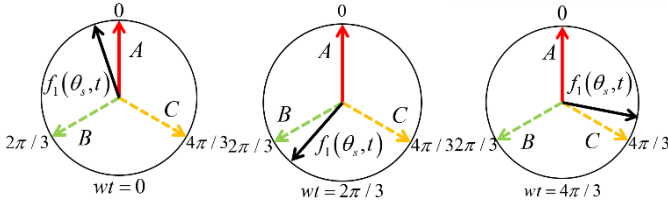


Fig. 2 Synthesized forward rotating magnetomotive force.

Applying a three-phase asymmetric current to an unequally toothed and slotted PMSM, the pulsating magnetic potential generated in the windings is decomposed as:

$$\begin{aligned} f_{A1} &= \frac{1}{2} F_{\phi 1} \cos(wt - \theta_s + b) + \frac{1}{2} F_{\phi 1} \cos(wt + \theta_s + b) \\ f_{B1} &= \frac{1}{2} F_{\phi 1} \cos(wt - \theta_s + \theta_b - \theta_B + b) + \frac{1}{2} F_{\phi 1} \cos(wt + \theta_s + \theta_b + \theta_B + b - 4\pi/3) \\ f_{C1} &= \frac{1}{2} F_{\phi 1} \cos(wt - \theta_s + \theta_c - \theta_C + b) + \frac{1}{2} F_{\phi 1} \cos(wt + \theta_s + \theta_c + \theta_C + b - 2\pi/3) \end{aligned} \quad (4)$$

From equation(4), it can be seen that when $\theta_B + \theta_b = 0$ and $\theta_C + \theta_c = 0$, the reverse rotating magnetomotive force is 0.

The stator magnetic permeability of the unequal tooth and slot PMSM is:

$$\begin{aligned} \Lambda_s(\theta, t) &= \Lambda_{s1}(\theta, t) + \Lambda_{s2}(\theta, t) \\ &= \Lambda_{s0} + \sum_{n=1,2,3,\dots} \Lambda_{nz} \cos(nn_w \theta) + \sum_{l=1,2,3,\dots} \Lambda_{lz} \cos(l(n_w + Z)\theta) \end{aligned} \quad (5)$$

The rotor magnetic permeability of the unequal tooth and slot PMSM is:

$$\Lambda_r(\theta, t) = \Lambda_{r0} + \sum_{m=1,2,3,\dots} \Lambda_{2m} \cos(2mp\theta - 2m\omega_1 t) \quad (6)$$

Applying three-phase symmetric currents, the magnetic flux density expression for the unequal tooth and slot PMSM is:

$$\begin{aligned} b_s(\theta, t) &= f_s(\theta, t) \cdot \lambda(\theta, t) \\ &= [f_{1p}(\theta, t) + f_{1n}(\theta, t)] \cdot [\Lambda_0 \cdot \Lambda_r(\theta, t) \cdot \Lambda_s(\theta, t)] \end{aligned} \quad (7)$$

III. WINDING CONFIGURATION SELECTION FOR UNEQUAL TOOTH AND SLOT PMSMS

Taking an 8-pole, 48-slot internal dual-width tooth PMSM as an example, the armature winding is distributed winding configuration. There are 6 types of winding configurations named as a, b, c, d, e, and f. The amplitudes of the rotating magnetomotive forces for the a-f winding configurations are presents in TABLE I.

TABLE I AMPLITUDES OF THE PMSMS' FORWARD AND REVERSE ROTATING MAGNETOMOTIVE FORCES

Types	Symmetrical current		Asymmetrical current	
	Forward	Reverse	Forward	Reverse
a	5.996	0.1492	5.985	0
b	5.979	0.1407	5.917	0
c	5.962	0.1578	5.849	0
d	5.945	0.1479	5.934	0
e	5.962	0.1578	5.934	0
f	5.979	0.1407	5.934	0

Different winding configurations affect the amplitudes of the forward and reverse rotating magnetomotive forces when symmetric currents are applied. Asymmetric currents can nullify the reverse rotating magnetomotive force but reduce the forward rotating magnetomotive force amplitude.

The torque of the dual-width tooth PMSMs is calculated using the finite element method. The output torque and torque ripple ratio of the motors vary with the current power angle, as shown in Fig. 3 and Fig. 4.

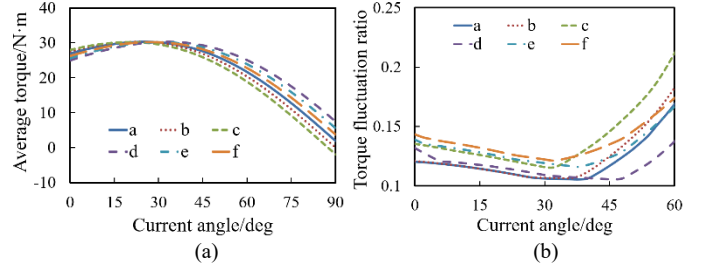


Fig. 3 Variation of (a) average torque and (b) torque ripple ratio with current power angle under three-phase symmetric currents.

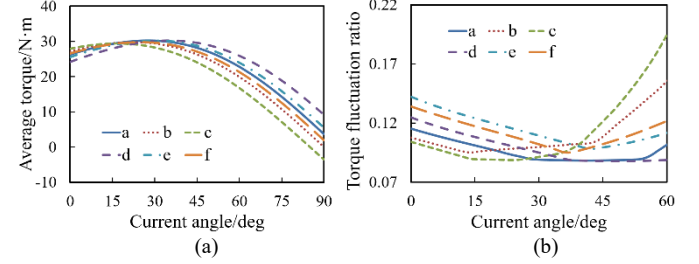


Fig. 4 Variation of (a) average torque and (b) torque ripple ratio with current power angle under three-phase asymmetric currents.

The simulation results from Fig. 3 and Fig. 4, along with TABLE I, validate that the amplitude and position of the synthesized forward rotating magnetomotive force affect the output torque, while the synthesized reverse rotating magnetomotive force impacts torque ripple. The maximum output torque of the 8-pole, 48-slot internal PMSM with equal tooth and slot occurs when the current power angle is 22.5°. For the internal dual-width tooth PMSM, applying a three-phase symmetric current with a power angle of 22.5° and using the b winding configuration results in the maximum torque and the minimum torque ripple.

IV. CONCLUSION

When three-phase symmetric currents are applied to unequal tooth and slot PMSMs, the forward magnetomotive forces cannot be directly summed, the reverse rotating magnetomotive forces cannot cancel out. Applying three-phase asymmetric currents can nullify the reverse rotating magnetomotive force. The amplitude and position of the synthesized forward rotating magnetomotive force in the unequal tooth and slot PMSM affect the torque, while the amplitude of the reverse rotating magnetomotive force influences the torque ripple. The winding configuration of the motor affects the synthesized magnetomotive force, and an appropriate winding configuration should be chosen during design.

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