

End Winding Inductance Computation Using a 2-Dimensional Approach

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This paper introduces an efficient two-dimensional (2D) method for calculating the leakage inductance of end windings in induction machines, traditionally a three-dimensional (3D) computational challenge. By leveraging a 2D H-formulation, the complex geometry of end windings is transformed into a simplified 2D model. Validation against 3D simulations and experimental results demonstrates very good agreement

Index Terms — End windings, inductance, H-formulation, induction machine.

I. INTRODUCTION

SINCE the 1940s, advancements in electrical machine design have been driven by the need for more accurate methods to solve the equations governing electrical phenomena. One of the most challenging aspects is addressing the magnetic effects of end windings, particularly in calculating their inductance.

Few researchers have developed fully numerical three-dimensional models for calculating the leakage inductance of end windings, which involve detailed geometric representations and three-dimensional formulation approaches [1]. These methods often demanded significant user input, leading to overly complex systems that were challenging to solve efficiently. In contrast, analytical methods have long been used to estimate the inductance of end windings in simpler cases [2]-[3].

In this paper, we propose a new two-dimensional approach to calculate the inductance of the end windings of induction machines. The proposed method transforms the three-dimensional geometry of end winding into a two-dimensional geometry using a 2D H-formulation [4] for the computation of the inductance.

The results obtained demonstrate excellent agreement with those derived from three-dimensional vector potential models [1] and experimental results.

II. METHOD USED

The method used to calculate the leakage inductance of coil overhangs requires simplifying the problem while keeping it as close to reality as possible. Fig. 1 illustrates the transformation from a 3D to a 2D representation that facilitates this simplification.

A. Transformation from 3D to 2D

Figure 1 illustrates the transformation from a 3D to a 2D representation that facilitates this simplification. In Fig. 1.a, the 3D model of the stator includes the magnetic core, winding slots, and a detailed representation of the coil overhang geometry. A radial cross-section of the stator is taken to isolate one end of the machine, focusing on the region where the end windings extend beyond the magnetic core. This section is then unfolded into a planar 2D representation, as shown in Fig. 1.b, preserving the geometry of the magnetic circuit (teeth and slots) and the spatial configuration of the coil overhangs. Finally, the 2D

model is re-wrapped into a circular layout, depicted in Fig. 1.c, positioning the end windings outside the magnetic circuit. To simplify the analysis, the three-dimensional magnetic circuit is eliminated in the 2D approach.

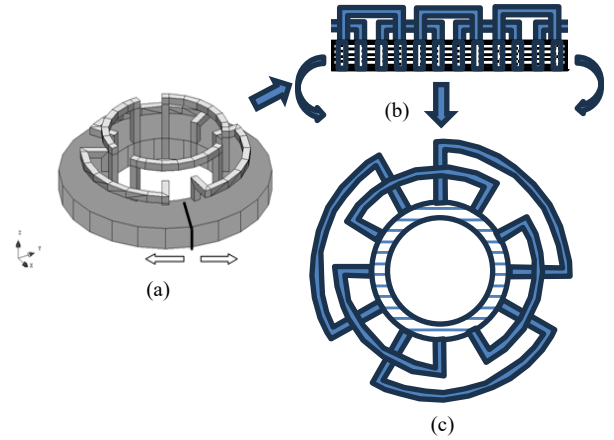


Fig. 1. End winding transformation geometry

This transformation reduces the complexity of the original 3D problem while retaining the essential geometric and electro-magnetic characteristics, enabling a computationally efficient analysis of the leakage inductance associated with the coil overhangs.

B. Formulation and Boundaries conditions

In this analysis, the H-formulation is considered to describe the electromagnetic behavior of the system. This formulation offers a more straightforward way to represent the interaction between the coil overhangs and the surrounding magnetic field within the simplified 2D domain. The governing equation for this formulation is expressed as:

$$\nabla \times \left(\frac{1}{\sigma} \nabla \times \vec{H}_z \right) + j\mu\omega \vec{H}_z = 0 \quad (1)$$

Where H_z is the z-component of the magnetic field (assuming a 2D geometry), σ is the electrical conductivity, μ is the magnetic permeability, and ω is the angular frequency of the applied field. This equation models the behavior of the magnetic field in the domain, incorporating both eddy current effects and the time-varying magnetic effects.

In this study, the magnetic field H_z is imposed on the boundaries of the coil overhangs to account for their interaction with

the surrounding magnetic circuit. Additionally, another boundary condition is applied to represent the distant boundary of the coil overhangs, ensuring that the magnetic field behavior in the far region is appropriately modeled. The analysis is conducted at a quasi-static frequency (near zero) to eliminate skin effects in the conductors, ensuring that the focus remains on calculating the leakage inductance. Fig. 2 shows a boundary condition applied to phase 1. For the other phases, the same condition is imposed but with a phase shift corresponding to the considered phase.

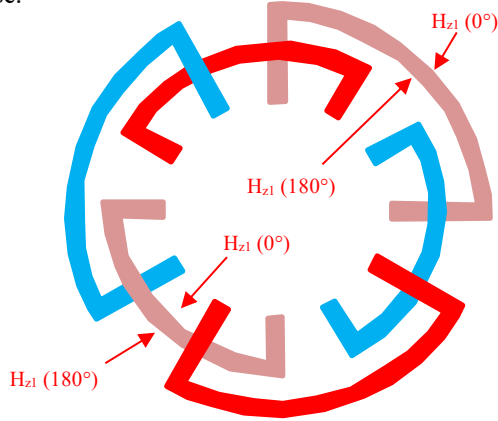


Fig. 2. 2D End winding and boundaries conditions

C. Leakage Inductance Calculation

The final step involves the calculation of leakage inductances associated with the end winding regions. The leakage inductance is calculated based on the energy quantity in the system using the following formula:

$$L = \frac{1}{I^2 \mu} \iiint_V B^2 dV \quad (2)$$

This includes integrating the local magnetic field and current density distributions to determine energy storage and associated inductive properties. These calculations provide a basis for comparing the simulated and measured results.

III. RESULTS

The machine studied is a real-world case: a bipolar three-phase motor operating at 50 Hz with a power rating of 4 kW. The stator of this motor has 24 slots, with 4 slots per pole per phase. The winding is pole-based with two winding paths. The calculations were performed using the Flux software in two dimensions.

This method focuses on calculating the inductance and the contribution of the coil overhangs, accounting solely for the frontal part of the machine. In contrast, the approach in reference [1] combines 3D calculations with a conventional 2D analysis using the magnetic vector potential, resulting in the total inductance of the machine, which is validated against experimental measurements. We adopted a similar approach, where the inductance calculated using the proposed method will be added to the inductance obtained through a traditional 2D analysis.

Fig. 3 shows the 3D geometry of the coil overhangs, and Fig. 4 shows the coil using the proposed 2D approach. The simulation results are recorded in Table 1.

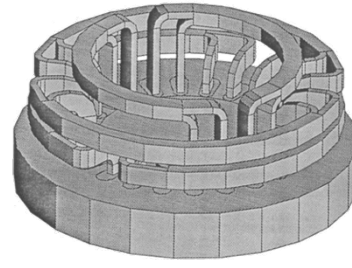


Fig. 3. 3D End winding geometry 3-phase induction machine

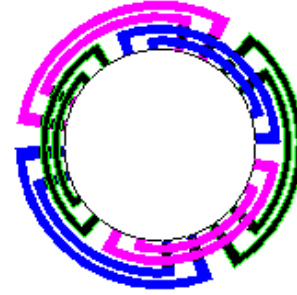


Fig. 4. 2D End winding geometry 3-phase induction machine

TABLE I
SIMULATION RESULTS

	2D	3D	Measured value
Leakage Inductance (mH)	4.49	4.29	4.02

IV. CONCLUSION

In this paper, a new 2D method for calculating the leakage inductance of end winding in induction machines is presented. By transforming the complex 3D geometry of the end windings into a simplified 2D model, the method significantly reduces computational time and memory requirements while maintaining high accuracy in the representation of electromagnetic behavior. The proposed approach, based on a 2D H-formulation, has been validated against 3D models and experimental measurements, demonstrating very good consistency with these benchmarks.

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