

Topology Optimization of Chip Inductor Using Density Method

Shuli Yin¹, Hajime Igarashi¹, and Stéphane Clénet²

¹Graduate School of Information of Technology, Hokkaido University, Japan, shuli.yin@polimi.it, igarashi@ssi.ist.hokudai.ac.jp

²Univ. Lille, Arts et Metiers Institute of Technology, EC Lille, Junia, L2EP Lille, France, stephane.clenet@ensam.eu

The digest proposes a gradient-based topology optimization method to design chip inductors. The density method is used to ensure that the material distribution is continuous and differentiable, and the adjoint variable method is employed to calculate efficiently the gradient of the objective function, combined with the finite element method. It has been verified that this method can efficiently design inductors that meet the target inductance value while minimizing Joule losses.

Index Terms—Adjoint variable method, density method, eddy currents, topology optimization.

I. INTRODUCTION

CHIP INDUCTORS typically have dimensions of the millimeter-scale, making them widely used in highly integrated electronic products [1]. Therefore, efficiently designing inductor topologies that meet requirements is crucial. Automatic design procedure based on topology optimization techniques can be really useful. Meanwhile, compared to topology optimization based in stochastic methods, gradient methods can be highly efficient. Therefore, in this study, the gradient-based density method to design and to optimize two-dimensional (2D) chip inductors is introduced. The aim is to reduce the Joule losses while keeping the value of the inductance. To represent the chip inductor, the finite element method (FEM) is applied, and we show how to derive the gradient using the adjoint variable method. The efficiency of the proposed method is tested through numerical experiments.

II. FORMULATIONS AND MODEL DESCRIPTION

We assume to design a 2D inductor with a cross-section of $2.4 \text{ mm} \times 1.2 \text{ mm}$, as shown in Fig. 1(a). On the left and right, there are symmetrical *Design region 1* and *Design region 2*, each measuring $1.0 \text{ mm} \times 1.0 \text{ mm}$, which respectively allow current to flow in and out. Additionally, each region is surrounded by an iron edge with a width (denoted by Δl) of 0.1 mm . Accounting for symmetry enables to optimize only half of the geometry (*Design region 1*).

The materials in *Design region 1* are either iron or copper. The conductivity σ and reluctivity ν of iron are 0 S/m and $7.96 \times 10^3 \text{ A}\cdot\text{m/Wb}$, respectively, while those of copper are $5.8 \times 10^7 \text{ S/m}$ and $7.96 \times 10^5 \text{ A}\cdot\text{m/Wb}$. To ensure the continuity and differentiability of material variations, we define the density ρ ($0 \leq \rho \leq 1$) in combination with the sigmoid function f to approximate σ or ν of materials [2]:

$$x(\rho) \approx x_{\text{iron}}(1 - f(\rho)) + x_{\text{cop}}f(\rho), \quad (1)$$

where x_{iron} represents the σ or ν of iron, while x_{cop} corresponds to the copper. As indicated by (1), the material is iron when ρ is 0 and copper when ρ is 1.

Following this, FEM is used to discretized the 2D magnetoquasistatic problem using the vector potential formulation. We obtain the following system of equations:

$$\mathbf{K}(\nu(\rho))\mathbf{A} + j\omega\mathbf{M}(\sigma(\rho))\mathbf{A} = \mathbf{V}/l_z, \quad (2)$$

where the vector ρ gathers the value of the density at each node of the mesh; $\mathbf{K}$ is the stiffness matrix which depends on the reluctivity ν given by (1) in function of the density; $\mathbf{M}$ is the mass matrix which depends on the conductivity σ also given by (1); $\mathbf{A}$ and $\mathbf{V}$ are vectors of the magnetic vector potential and the voltage source on each node respectively; the vector ρ gathers the value of the density at each node of the mesh. l_z is the length in z -direction and set to be 1 m for convenience, ω stands for the angular frequency. With the application of the boundary conditions shown in Fig. 1 (a), and the total external voltage source V in Fig. 1 (b) known, $\mathbf{A}$ can be solved. And based on this, the stored energy W_{co} and the Joule losses J_{loss} can also be determined using:

$$W_{co} = \mathbf{A}^t \cdot \mathbf{K} \cdot \mathbf{A}^* / 2, \quad (3)$$

$$J_{loss} = (-j\omega\mathbf{A} + \mathbf{V})^t \cdot \mathbf{M} \cdot (-j\omega\mathbf{A} + \mathbf{V})^*. \quad (4)$$

Then the value of inductance L is derived to be:

$$L = 2W_{co}V^2 / (4J_{loss}^2 + 16\omega^2W_{co}^2). \quad (5)$$

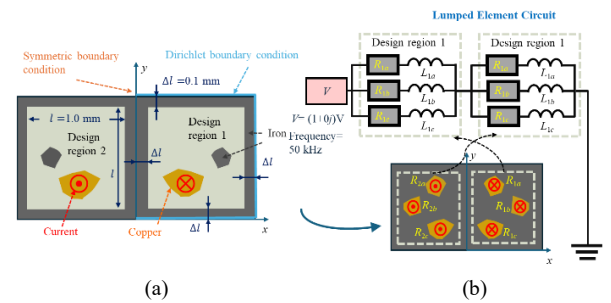


Fig. 1. Illustration of the 2D inductor to be optimized, and its lumped circuit diagram. (a) Geometric structure and boundary conditions. (b) the lumped external electrical circuit.

After obtaining the required characteristic parameters, we can begin to introduce the optimization process. In this study, our design goal is to ensure that the value of L meets the design target L_{tar} , while minimizing J_{loss} . This can be expressed using the objective function F , which satisfies:

$$F = W_1 \frac{(L - L_{tar})^2}{L_{tar}^2} + W_2 J_{loss} \rightarrow \min, \quad (6)$$

in which W_1 and W_2 are the corresponding weighting coefficients. To minimize F , the gradient-based method will be used. Subsequently, we proceed to compute the sensitivity of F . To avoid the challenging term of $\partial A / \partial \rho_k$ (ρ_k is the value of the density at the node k) that arises when directly computing the derivative of F , we use the adjoint variable method which consists in considering an approximation $\hat{F}$:

$$\hat{F} \approx F + \mathbf{Z}^t \cdot [(\mathbf{K} + j\omega\mathbf{M})\mathbf{A} - \mathbf{V}], \quad (7)$$

where $\mathbf{Z}$ is an auxiliary vector and can be solved using [3]:

$$(\mathbf{K} + j\omega\mathbf{M})^t \mathbf{Z} = \frac{\partial F}{\partial \mathbf{A}_r} - \frac{\partial F}{\partial \mathbf{A}_i} j, \quad (8)$$

in which the subscript r and i refer to the real and imaginary parts of $\mathbf{A}$, respectively. The right hand of (8) can be obtained analytically through (3)-(5). Once $\mathbf{Z}$ is obtained, the sensitivity of F can be approximated as [2]

$$\frac{\partial F}{\partial \rho_k} \approx \frac{\partial \hat{F}}{\partial \rho_k} = -\text{Re} \left(\mathbf{Z}^t \frac{\partial (\mathbf{K} + j\omega\mathbf{M})}{\partial \rho_k} \mathbf{A} \right). \quad (9)$$

The partial derivatives of $\mathbf{K}$ and $\mathbf{M}$ can be also obtained explicitly. Once the sensitivity is solved, and using the ρ_k obtained from the (p) -th iteration, the new density distribution at the $(p+1)$ -th iteration can be finally obtained using L-BFGS-B method [4]:

$$\rho_k^{(p+1)} = \rho_k^{(p)} - \eta^{(p+1)} \mathbf{H}_k \frac{\partial F}{\partial \rho^{(p)}}. \quad (10)$$

In (10), η is the step size, determined through line search, and $\mathbf{H}_k$ is the k -th row of the approximate inverse Hessian matrix [4] obtained through previous gradients and positions.

III. OPTIMIZATION RESULTS

We want to design a 2D inductor with $L_{tar}=8.0 \mu\text{H}$ minimizing Joule losses. The coefficients W_1 and W_2 in (6) are taken equal to 60.0 and 50.0 respectively. The frequency of the voltage source is 50 kHz, The skin depth is equal to 0.3 mm for the copper much smaller than the dimension of the design region. The eddy current effect cannot be negligible. After 69 iterations, the optimal design that meets the specified requirements is successfully obtained, with the topology of *Design region 1* shown in Fig. 2. Additionally, design solutions at different stages of the iteration process are also presented.

In addition, the details of the convergency history are also given in Fig. 3. It can be observed that F stabilizes after only 69 iterations. Meanwhile, the inductance gradually approaches the target design value with the minimum error being only 1.8%, and the Joule loss decreases progressively until it stabilizes.

Despite the optimal solution is obtained, the shape is based on the fictitious composite solution. Therefore, in the optimal design, elements with $\rho < 0.5$ are categorized as iron, while those with $\rho \geq 0.5$ are categorized as copper. After the classification of materials, finally the topology structure and its field distribution are presented in Fig. 4. The inductance value is $7.86 \mu\text{H}$, and the Joule losses value is 0.013 W , which are

very close to the optimal fictitious composite solution obtained with a continuous variation of the density.

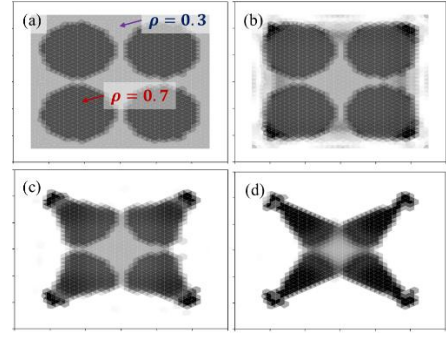


Fig. 2. The grayscale representation of design solutions (the density ρ varies from 0 to 1), where black corresponds to copper and white represents iron. (a) Initial shape. (b) Design at the 5th iteration. (c) Design after the 15th iteration. (d) Final optimal solution.

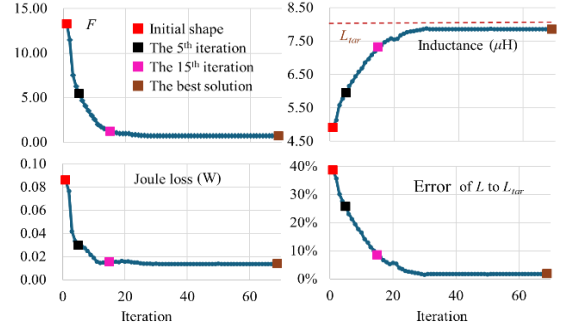


Fig. 3. Convergence history and relative error of L with respect to L_{tar} .

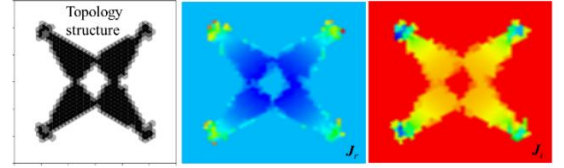


Fig. 4. Topology structure after the category, and the distribution of current density $\mathbf{J}$ for the real and imaginary part.

IV. CONCLUSION

The adjoint variable method combined with the density method is proved to be powerful for topology optimization on the chip inductors when considering both the inductance and the Joule loss. The full paper will provide more derivation process and discussion.

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