

Nonlinear Reduced Order Modeling of Cage Induction Motors Using Cauer Ladder Network and Time-Harmonic Finite-Element Analysis

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This study investigates model order reduction of a cage induction motor (IM) based on the multiport Cauer ladder network (CLN) method considering nonlinear magnetic properties. First, the permeability distributions in the stator and rotor are obtained using nonlinear time-harmonic eddy-current analysis as a function of the primary current and the slip. Then, the circuit parameters of the parametrized multiport Cauer circuits for the stator and rotor are derived under the conditions of the calculated permeability distributions as the nonlinear model order reduction of the cage IM. The effectiveness of the proposed method is validated by comparing the numerical results obtained by the CLN method and a finite-element method.

Index Terms—cage induction motor, Cauer ladder network method, finite-element method, model order reduction, nonlinear magnetic property, time-harmonic analysis.

I. INTRODUCTION

RECENTLY, model order reduction (MOR) for a cage induction motor (IM) has been extensively studied [1]–[2], and the Cauer ladder network (CLN) method [3] is one of promising MOR approaches. In applying the CLN method to a cage IM, the stator and rotor domains are divided in the air gap, and equivalent Cauer circuits are derived separately in each domain. These equivalent circuits are then connected based on the boundary conditions for the space harmonics (SHs) included in the air-gap flux density waveform [4]. For practical application to cage IMs, a method for selecting dominant SHs in the air-gap flux density waveform [5], a modeling of the cage IM with skewed rotor slots [6], and a nonlinear MOR based on parameterized multiport CLN method [7] were investigated. In [7], however, the nonlinear magnetic properties are considered only in the stator. In the closed rotor slots, it is essential to take account of the nonlinear magnetic properties of the rotor core because it generally has a great influence on the performance of the cage IM.

Based on this background, in this study, the CLN method for cage IM, which considers the nonlinear magnetic properties in the stator and rotor domains is proposed. When calculating the circuit parameters of the CLN, the permeability distributions in the stator and rotor are obtained using nonlinear time-harmonic eddy-current analysis [8] as a function of the primary current and the slip. Then, the circuit parameters of the parameterized multiport Cauer circuits for the stator and rotor are derived based on the calculated permeability distributions. The effectiveness of the proposed method is validated through the analysis of a cage IM with closed rotor slots by the CLN method and a finite-element method (FEM).

II. MULTIPORT CLN METHOD FOR CAGE IM CONSIDERING NONLINEAR MAGNETIC PROPERTIES

We analyze the cage IM with closed rotor slots shown in Fig. 1. Fig. 2 shows the average torque obtained by linear time-

harmonic eddy-current analysis (LTHA), nonlinear time-harmonic eddy-current analysis (NLTHA), and time-stepping analysis (TSA) when applying the rated current while keeping the slip frequency sf constant and varying the power supply frequency f . In general, the closed rotor slots have a great influence on the motor performance because the tip of the rotor tooth is highly saturated. Therefore, the numerical results of the LTHA differ significantly from those of the TSA regardless of the value of sf . In contrast, the NLTHA has sufficiently good accuracy compared with the TSA. This is because the effect of the slot harmonics on the torque is relatively small due to the closed rotor slots. Since the motor performance does not vary under the condition of constant sf , the reduced order model of the cage IM with closed rotor slots, which is significantly affected by the nonlinear magnetic properties of the stator and rotor cores, can be represented by the function of the RMS value of the primary current i_e and the slip s or f .

Here, we propose to derive the parameterized CLN for cage IMs, using the NLTHA. Fig. 3 shows the procedure of the proposed method. First, the permeability distributions are calculated using the time-harmonic eddy-current analysis while varying i_e and s . Then, the circuit parameters of the parametrized CLNs are obtained based on the calculated permeability distributions as a function of i_e and s in this study. Since we can obtain the permeability distributions corresponding to each current and slip condition in a single NLTHA, the proposed approach minimizes the increase in the computational cost required to determine the circuit parameters of the parametrized CLNs.

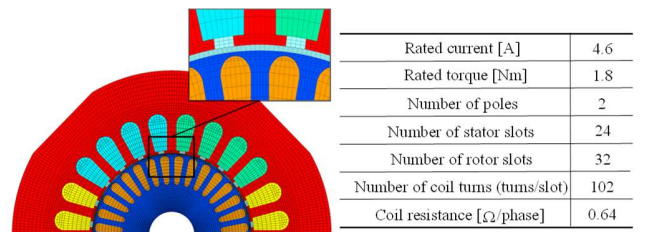


Fig. 1. Mesh of cage IM with closed rotor slots

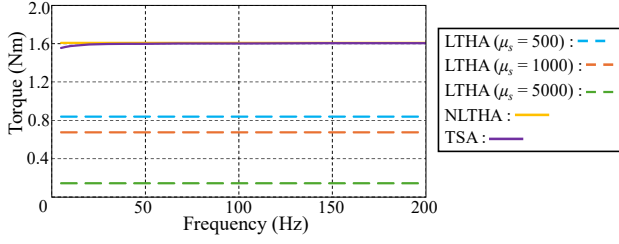


Fig. 2. Torque characteristics under constant slip frequency condition.

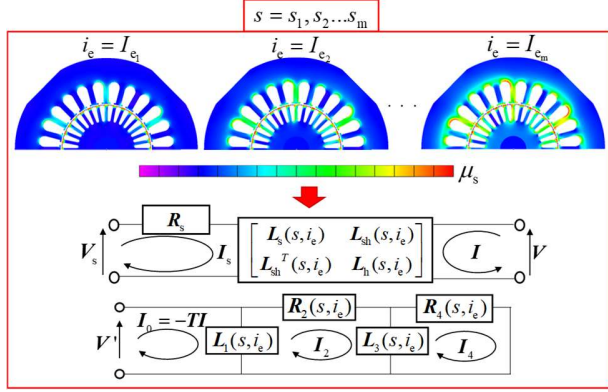


Fig. 3. Procedure of nonlinear CLN method based on time-harmonic analysis.

III. NUMERICAL RESULTS

The effectiveness of the proposed method is verified in the analysis of the cage IM shown in Fig. 1. The dominant SHs in the air-gap flux density are determined by the time and space harmonic analysis method in frequency domain [9] at $s = 0$. Fig. 4 shows the amplitude of the time and space harmonics larger than 1 % of the fundamental. The dominant SHs are mainly caused by the stator slots. The SHs due to the rotor structure are negligibly small because the rotor has closed slots. In this study, the SHs larger than 5 % of the fundamental were selected as the dominant components. The number of selected SHs for the CLN method is 5.

Fig. 5 shows the numerical results of the U-phase current and torque obtained from the FEM, conventional linear CLN and nonlinear CLN at $s = 1/32$. In the linear CLN, the relative permeability of the iron core was set to 1000. The numerical results differ significantly between the FEM and the linear CLN because the consideration of nonlinear magnetic properties is essentially required in the cage IM with closed rotor slots. By contrast, the U-phase current and torque waveforms obtained from the nonlinear CLN are in good agreement with those obtained from the FEM, although the pulsations are not exactly represented. Table I lists the computation time required to extract the dominant SHs, derive the circuit parameters, and conduct the transient circuit analysis at $s = 1/32$. The value in parentheses indicates the number of periods required to obtain the steady-state solutions. The proposed method is much faster than the FEM, although it needs more calculation time than the linear CLN for deriving the circuit parameters. This is because the nonlinear CLN requires several NLTHAs to determine the current-dependent circuit parameters. The results confirm the effectiveness of the proposed nonlinear CLN from the perspectives of computational accuracy and cost. The details of

the proposed approach and more numerical results will be included in the full paper.

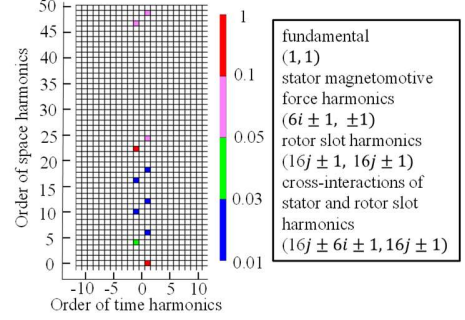


Fig. 4. Results of time and space harmonic analysis of air-gap flux density.

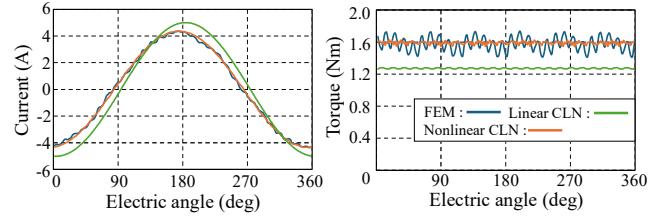


Fig. 5. Comparison of numerical results obtained by FEM and CLN method.

TABLE I
COMPUTATIONAL TIME

Analysis condition	Computational time [s]		
	Extraction of space harmonics	Derivation of circuit parameters	Transient analysis at $s = 1/32$
NLCLN	6.73	226.2	0.22 (10)
LCLN	-	17.1	0.17 (10)
FEM	-	-	689.2 (4)

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