

Boundary Element Modeling of Magnetoelectric Anisotropic Materials

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We present a method for modeling arbitrarily shaped anisotropic magnetoelectric objects immersed in a homogeneous isotropic medium and exposed to an arbitrary electric field. The method requires the discretization of only boundary layers and solves the problem directly, without transforming it into an isotropic one. We investigate anisotropic magnetoelectric materials of the Tellegen type, characterized by nine parameters for each of the permittivity, permeability, and coupling matrices. Results are compared against an analytical solution for the case of a magnetoelectric anisotropic sphere placed in air and exposed to a uniform electric field. We achieve a mean relative error within the sphere below 1.4% and a normalized root mean square error outside the sphere below 0.1%. With a slight modification, the method can be applied to magnetoelectric materials exposed to a magnetic or combined electric and magnetic fields.

Index Terms—Anisotropy, boundary element methods, magnetoelectric effect.

I. INTRODUCTION

MAGNETOELECTRIC MATERIALS can induce magnetic polarization upon the application of an electric field, while electric polarization can be tuned by an external magnetic field [1]. This property has opened up a wide range of applications for these materials. They have already been tested for wireless neural stimulation [2], cancer treatments [3], wireless energy transfer [4], implantable bioelectronics [5], and many more.

To develop new devices where the magnetic properties of the magnetoelectric materials and consequently their deformations can be easily controlled by an electric field, effective numerical calculations are necessary. Simple and regular geometries can be analyzed analytically [6]. For arbitrarily shaped geometries, the finite element method (FEM) is typically employed [7]. However, the FEM necessitates discretization of the entire volume, making it computationally expensive. Reducing the dimensionality of the discretization can be achieved using the boundary element method (BEM). However, due to the inability to express bi-anisotropic fundamental solutions in an explicit closed form, a standard formulation of the BEM for bi-anisotropic media has not yet been established.

We introduce a formulation based on the boundary element source method (BESM) that can model an arbitrarily shaped bi-anisotropic object embedded in an isotropic background medium and exposed to an external electric field. We uniformly spread both polarized electric and fictitious magnetic charges over the surfaces of boundary elements and calculate their surface densities to satisfy boundary conditions for the normal components of the electric displacement and the magnetic flux density. Definitions of anisotropic tensors, magnetoelectric coupling, and the methodology itself are provided in Section II. A simple numerical example of a bi-anisotropic sphere is considered in Section III and verified against an analytical solution in the same section. Section IV provides conclusions and opportunities for further development of the method.

II. METHODOLOGY

A. Anisotropic magnetoelectric tensors

We consider a bi-anisotropic object with permittivity tensor $\overset{\leftrightarrow}{\epsilon}_1$, permeability tensor $\overset{\leftrightarrow}{\mu}_1$, and coupling tensor $\overset{\leftrightarrow}{\xi}_1$, immersed in an isotropic medium with a permittivity ϵ_2 and permeability μ_2 , and exposed to an external homogeneous electric field $\mathbf{E}_0$, as shown in Fig. 1. We represent the second-order tensors

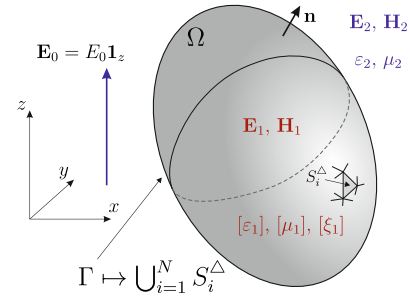


Fig. 1. An arbitrarily shaped bi-anisotropic object of a permittivity matrix $[\epsilon_1]$, permeability matrix $[\mu_1]$, and a coupling coefficient matrix $[\xi_1]$, embedded in an isotropic medium ϵ_2 , μ_2 and exposed to a homogeneous electric field $\mathbf{E}_0$.

$\overset{\leftrightarrow}{\epsilon}_1$, $\overset{\leftrightarrow}{\mu}_1$, and $\overset{\leftrightarrow}{\xi}_1$ by the square matrices $[\epsilon_1]$, $[\mu_1]$, and $[\xi_1]$, respectively.

The bi-anisotropic object is of the Tellegen type, where the electric displacement vector $\mathbf{D}_1$ and the magnetic flux density vector $\mathbf{B}_1$ are coupled through the following relations:

$$\mathbf{D}_1 = [\epsilon_1]\mathbf{E}_1 + [\xi_1]\mathbf{H}_1, \quad (1)$$

$$\mathbf{B}_1 = [\mu_1]\mathbf{H}_1 + [\xi_1]\mathbf{E}_1. \quad (2)$$

B. Boundary Element Source Method

At the boundary surface Γ , as shown in Fig. 1, the normal components of the electric displacement vector and the mag-

netic flux density vector remain unchanged: $\mathbf{D}_1 \cdot \mathbf{n} = \mathbf{D}_2 \cdot \mathbf{n}$, $\mathbf{B}_1 \cdot \mathbf{n} = \mathbf{B}_2 \cdot \mathbf{n}$. Using (1) and (2), we obtain:

$$\mathbf{E}_2 \cdot \mathbf{n} = ([\varepsilon_1]\mathbf{E}_1 + [\xi_1]\mathbf{H}_1) \cdot \mathbf{n} / \varepsilon_2, \quad (3)$$

$$\mathbf{H}_2 \cdot \mathbf{n} = ([\mu_1]\mathbf{H}_1 + [\xi_1]\mathbf{E}_1) \cdot \mathbf{n} / \mu_2. \quad (4)$$

At the boundary Γ , changes in the normal components of the electric and magnetic polarization vectors $\mathbf{P}_e = \mathbf{D} - \varepsilon_0 \mathbf{E}$ and $\mathbf{P}_m = \mathbf{B} - \mu_0 \mathbf{H}$, are produced by the bounded electric charge density η_e and the fictitious magnetic charge density η_m , which are given by

$$\eta_e = \varepsilon_0 [([\varepsilon_1]/\varepsilon_2 - [\mathbf{I}]) \mathbf{E}_1 + [\xi_1]\mathbf{H}_1/\varepsilon_2] \cdot \mathbf{n}, \quad (5)$$

$$\eta_m = \mu_0 [([\mu_1]/\mu_2 - [\mathbf{I}]) \mathbf{H}_1 + [\xi_1]\mathbf{E}_1/\mu_2] \cdot \mathbf{n}. \quad (6)$$

The total electric field $\mathbf{E}_1$ and magnetic field $\mathbf{H}_1$ are calculated at the centroids of the boundary elements on the inner side of the boundary Γ ($\Gamma \mapsto \bigcup_{i=1}^N S_i^\Delta$). Coulomb's law is used to calculate the electric fields at the centroids of all triangles excluding the triangle itself. The electric field at the centroid of the element S_i^Δ itself is calculated using an analytical formula for a uniformly charged triangle [8]. For the magnetic field calculation, the electric charge is replaced by a magnetic charge, and the permittivity ε_0 by the permeability μ_0 . Applying (5) and (6) to N boundary elements results in a system of $2N$ equations with N unknown bounded electric charges and the N unknown fictitious magnetic charges. Solving this system determines all the charges. The electric and magnetic fields at any point are a superposition of the primary field $\mathbf{E}_0$ and the secondary fields generated by the electric and fictitious charges at the boundary.

C. Testing Model

We consider an anisotropic magnetoelectric sphere with a radius $a = 1$ cm placed in air and subjected to a homogeneous electric field $\mathbf{E}_0 = E_0 \mathbf{1}_z$, $E_0 = 1$ V/m. The choice of the z -direction for the field does not imply any loss of generality. Since magnetoelectric materials are typically described by symmetric matrices, which can always be diagonalized [9], we use the following permittivity and permeability matrices:

$$[\varepsilon_1] = \varepsilon_0 \text{diag}(5, 8, 9), \quad [\mu_1] = \mu_0 \text{diag}(3, 6, 7),$$

and a scalar coupling coefficient $\xi_1 = \sqrt{p\varepsilon_{11}\mu_{11}}$ with the coupling factor $p = 0.5$.

D. Error Estimation

The BESM results are compared against a reference analytical solution [6] on a regular grid of points generated with a 2 mm step within a cuboid of $4a$ edge length. For points outside a sphere ($N_o = 8733$), we calculate the normalized root mean square deviation $\epsilon_z^\%$:

$$\epsilon_z^\% = \frac{\sqrt{\frac{1}{N_o} \sum_{i=1}^{N_o} (E_{z,i}^{\text{BESM}} - E_{z,i}^{\text{ANA}})^2}}{\max_{i=1 \dots N_o} E_{z,i}^{\text{ANA}} - \min_{i=1 \dots N_o} E_{z,i}^{\text{ANA}}} 100\% \quad (7)$$

and the mean relative error $\delta_z^\%$

$$\delta_z^\% = \text{mean}_{i=1 \dots N_i} \left| \frac{E_{z,i}^{\text{BESM}} - E_{z,i}^{\text{ANA}}}{E_{z,i}^{\text{ANA}}} \right| 100\% \quad (8)$$

associated with the standard deviation $\sigma_\%$ for the points lying inside the sphere ($N_i = 456$).

III. RESULTS AND DISCUSSION

Table I presents mean relative error and the SD for the points inside the sphere and the NRMSD outside the sphere for the z -components of the electric and magnetic field vectors.

TABLE I
MEAN RELATIVE ERROR AND NRMSD FOR TEST EXAMPLE

	$\delta_z^\% (\sigma_\%)$ (inside sphere)	$\epsilon_z^\%$ (outside sphere)
E_z	1.380 (0.157)	0.088
H_z	0.453 (0.289)	0.050

The mean relative error inside the sphere is less than 1.4 % and the NRMSD of the field outside the sphere is less than 0.1 %. The results demonstrate a very good agreement with the analytical solution.

IV. CONCLUSION

Bi-anisotropic materials, when subjected to magnetic or electric field, exhibit both polarization and magnetization. This unique characteristic offers significant advantages and a wide range of applications for these materials. We have modeled bi-anisotropic objects placed in an external electric field using solely boundary elements. The number and shape of these objects can be arbitrary. The source of the primary field can originate either inside or outside the model. Furthermore, method can be easily modified to account for a background magnetic field.

REFERENCES

- [1] W. Eerenstein, N. D. Mathur, and J. F. Scott, "Multiferroic and magnetoelectric materials," *Nature*, vol. 442, pp. 759-765, Aug. 2006.
- [2] A. Singer, S. Dutta, E. Lewis, Z. Chen, J. C. Chen, N. Verma, B. Avants, A. K. Feldman, J. O'Malley, M. Beierlein, C. Kemere, and J. T. Robinson, "Magnetoelectric Materials for Miniature, Wireless Neural Stimulation at Therapeutic Frequencies," *Neuron*, vol. 107, no. 4, pp. 631-643.e5, Aug. 2020.
- [3] X. Kong, P. Gao, J. Wang, Y. Fang, and K. C. Hwang, "Advances of medical nanorobots for future cancer treatments," *Journal of Hematology and Oncology*, vol. 16, paper no. 74, July 2020.
- [4] A. Kumar, S. Newacheck, and G. Youssef, "Parametric investigations of wireless energy transfer using strain-mediated magnetoelectric transmitter-receiver," *Smart Materials and Structures*, vol. 32, no. 11, paper no. 115023, Oct. 2023.
- [5] F. Alrashdan, K. Yang, and J. T. Robinson, "Magnetoelectrics for Implantable Bioelectronics: Progress to Date," *Accounts of Chemical Research*, vol. 57, no. 20, pp. 2953-2962, Oct. 2024.
- [6] A. Cherepanov and A. Sihvola, "Internal electric field of anisotropic and bianisotropic spheres," *Journal of Electromagnetic Waves and Applications*, vol. 10, no. 1, pp. 79-91, 1996.
- [7] J. Zhang, X. Wang, X. Chen, H. Du, and G. J. Weng, "Finite element analysis of the magnetoelectric effect on hybrid magnetoelectric composites," *Composite Structures*, vol. 296, paper no. 115876, Sep. 2022.
- [8] S. Rao, A. Glisson, D. Wilton, and B. Vidula, "A simple numerical solution procedure for static problems involving arbitrary-shaped surfaces," *IEEE Transactions on Antennas and Propagation*, vol. 27, no. 5, pp. 604-608, Sep. 1979.
- [9] O. Aberth, "The Transformation of Tensors into Diagonal Form," *SIAM Journal on Applied Mathematics*, vol. 15, no. 5, pp. 1247-1252, Sep. 1967.