

Time-Marching Scheme with Auxiliary Variables for Darwin Model in Electromagnetic Finite Element Analysis

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This paper proposes a method for time-domain analysis using the Darwin model. In this approach, the second time derivative term is removed using the redundant variables to apply the standard backward Euler method. We analyze a parallel plate capacitor model and compare the results with the frequency-domain analysis, thereby demonstrating the validity of the proposed method.

Index Terms— $A - \phi$ formulation, Darwin model, Finite-element methods, Time-domain analysis.

I. INTRODUCTION

The impact of parasitic capacitance and inductance is increasingly significant due to the recent miniaturization of electronic devices, which demands higher switching frequencies. The finite element analysis utilizing the Darwin model has been proposed to address both capacitance and inductance components effectively [1], [2], [3].

The Darwin model overcomes the drawbacks of the full-wave and magnetoquasistatic models, the two conventional methods of the $A - \phi$ formulation for electromagnetic finite element analysis. In the full-wave model, we utilize the Maxwell equations without neglecting the displacement current; consequently, the second time derivative term appears, which can make the calculation in the time domain challenging. The magnetoquasistatic model is widely used as an approximation of the Maxwell equations in which the second time derivative term does not appear due to neglecting the displacement current term. However, we cannot consider capacitance components for the model to be used only in a low-frequency range. The Darwin model, in contrast to these models, considers only the Coulomb field component of displacement current, which allows it to deal with the capacitance effect.

Several studies concerning the frequency-domain analysis employing the Darwin model have been conducted [3], [4], [5]. However, the time-domain analysis [6], [7] remains insufficiently discussed. Therefore, this article proposes a method of time-domain analysis based on the Darwin model formulation in [5], [6]. In the reference [6], the treatment of the second time derivative term was based on the central difference approximation; however, the discretization method for time derivatives was not studied in detail. In this paper, we demonstrate that an expression not including the second time derivative term can be derived with the redundant variables, to realize the time-discretization by the standard backward Euler method.

II. DARWIN MODEL

The Darwin model only considers the Coulomb field component of the displacement current. Utilizing the magnetic vector

potential $\mathbf{A}$ and the electric scalar potential ϕ , the $A - \phi$ formulation of the Darwin model is expressed as follows:

$$\nabla \times (\nu \nabla \times \mathbf{A}) + \sigma \left(\frac{\partial \mathbf{A}}{\partial t} + \nabla \phi \right) + \frac{\partial}{\partial t} \varepsilon \nabla \phi = \mathbf{J}_s, \quad (1)$$

$$\nabla \cdot \left[\sigma \left(\frac{\partial \mathbf{A}}{\partial t} + \nabla \phi \right) + \frac{\partial}{\partial t} \varepsilon \left(\frac{\partial \mathbf{A}}{\partial t} + \nabla \phi \right) \right] = 0, \quad (2)$$

where ν denotes magnetic reluctivity, ε denotes permittivity, σ denotes electrical conductivity, and $\mathbf{J}_s$ denotes source-current density. Note that the Coulomb-type gauge condition

$$\nabla \cdot \varepsilon \frac{\partial \mathbf{A}}{\partial t} = 0 \quad (3)$$

is implicitly imposed by (1) and (2).

Expressing $\frac{\partial}{\partial t} = \partial_t$, the finite element discretization of (1) and (2) is given as follows:

$$\begin{bmatrix} C^T [\nu] C + \partial_t [\sigma] & ([\sigma] + \partial_t [\varepsilon]) G \\ \partial_t G^T ([\sigma] + \partial_t [\varepsilon]) & G^T ([\sigma] + \partial_t [\varepsilon]) G \end{bmatrix} \begin{bmatrix} \mathbf{a} \\ \phi \end{bmatrix} = \begin{bmatrix} \mathbf{b} \\ 0 \end{bmatrix}. \quad (4)$$

In (4), $\mathbf{a}$ and ϕ represent the discrete vectors corresponding to $\mathbf{A}$ and ϕ , respectively. C denotes the discrete rotation matrix, and G denotes the discrete gradient matrix. The discrete finite element matrices $[\nu]$, $[\sigma]$, and $[\varepsilon]$ are related to magnetic reluctivity, electrical conductivity, and permittivity, respectively.

Since (4) is an extremely ill-conditioned system of equations [5], we introduce redundant variables χ [4] defined by

$$\mathbf{A} = \mathbf{A}' + \nabla \int \chi dt, \quad (5)$$

$$\phi = \phi' - \chi \quad (6)$$

to improve convergence characteristics. Consequently, we obtain

$$\begin{bmatrix} C^T [\nu] C + \partial_t [\sigma] & ([\sigma] + \partial_t [\varepsilon]) G & -\partial_t [\varepsilon] G \\ \partial_t G^T ([\sigma] + \partial_t [\varepsilon]) & G^T ([\sigma] + \partial_t [\varepsilon]) G & 0 \\ -\partial_t G^T [\varepsilon] & 0 & -G^T [\varepsilon] G \end{bmatrix} \begin{bmatrix} \mathbf{a}' \\ \phi' \\ \chi \end{bmatrix} = \begin{bmatrix} \mathbf{b} \\ 0 \\ 0 \end{bmatrix}, \quad (7)$$

where χ denotes the discrete vector corresponding to χ .

III. TIME-DOMAIN ANALYSIS

The matrix equations of the Darwin model (7) include the term $(\partial_t)^2$, which is the second time derivative term, thereby complicating the time-domain analysis. Therefore, by adding the third row of (7) multiplied by ∂_t to the second row, we obtain

$$\begin{bmatrix} C^T [\nu] C + \partial_t [\sigma] & ([\sigma] + \partial_t [\varepsilon]) G & -\partial_t [\varepsilon] G \\ \partial_t G^T [\sigma] & G^T ([\sigma] + \partial_t [\varepsilon]) G & -\partial_t G^T [\varepsilon] G \\ -\partial_t G^T [\varepsilon] & 0 & -G^T [\varepsilon] G \end{bmatrix} \begin{bmatrix} \mathbf{a}' \\ \phi' \\ \chi \end{bmatrix} = \begin{bmatrix} \mathbf{b} \\ 0 \\ 0 \end{bmatrix}. \quad (8)$$

The term of $(\partial_t)^2$ can be deleted by this transformation.

When (8) is expressed as

$$(K + \partial_t M) \mathbf{x} = \mathbf{b}, \quad (9)$$

using real matrices K and M , by defining time step Δt and applying the backward Euler method to this equation, we obtain the time-marching scheme:

$$K \mathbf{x}_n + M \frac{\mathbf{x}_n - \mathbf{x}_{n-1}}{\Delta t} = \mathbf{b}_n, \quad (10)$$

where $\mathbf{x}_n$ and $\mathbf{b}_n$ represent unknown vector $\mathbf{x}$ and right-hand-side vector $\mathbf{b}$ on the time $t = n\Delta t$, respectively. In our numerical experiments, the computation according to (10) was stable even when the time step was increased.

IV. NUMERICAL RESULTS

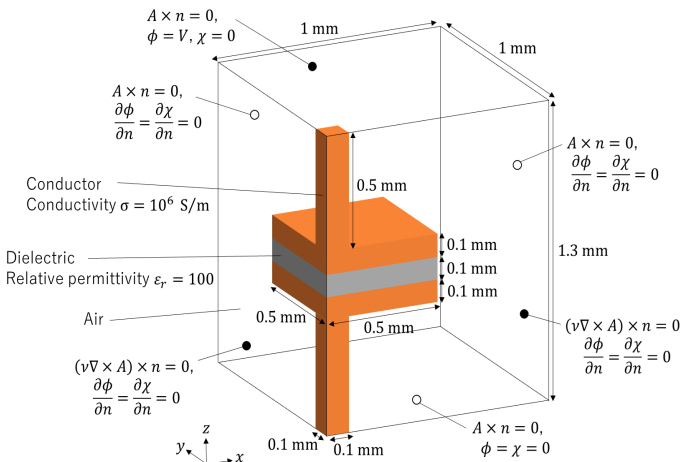


Fig. 1. Parallel plate capacitor model

We apply the time-domain analysis to the parallel plate capacitor model using the Darwin model, as illustrated in Fig. 1. We employ the Incomplete LU (ILU) preconditioner and the generalized minimal residual method (GMRES) for solving (10). Fig. 2 illustrates the current flowing through the bottom of the capacitor model when a 10MHz sinusoidal voltage is applied to it.

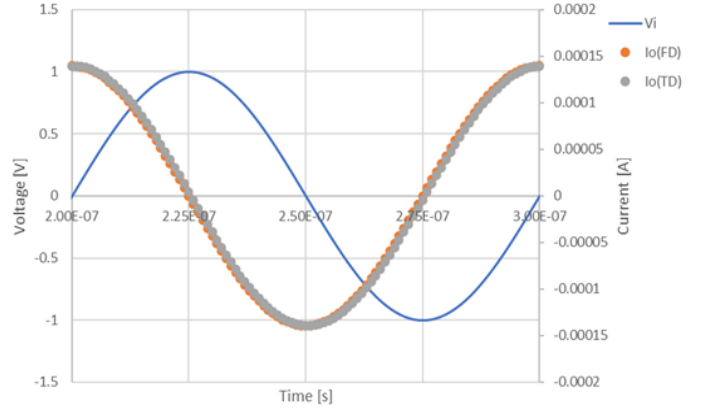


Fig. 2. Input voltage V_i (blue), output current $I_o(TD)$ (gray) that is computed by (10), and $I_o(FD)$ (orange) that denotes the result of frequency-domain analysis

Fig. 2 exhibits the phase of output current advances by $\frac{\pi}{2}$ relative to the input voltage in both the frequency- and time-domain analysis. This numerical test confirmed that the proposed scheme properly simulated the capacitance effect.

V. CONCLUSION

In this study, we proposed a time-domain analysis method using the Darwin model and ensured its validity using numerical simulation. In this short paper, we showed only results of the simple capacitor model due to space limitation issues. Results of a more complicated model will be showed in the full paper.

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