

Electro-Magnetic Decoupling Preconditioner for Solving Eddy Current Problems with Current and Voltage Excitations

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The evaluation of eddy current losses in windings is a significant task for designing efficient electric machines. Previous research has highlighted challenges with iterative methods for solving linear systems in eddy current problems with current and voltage excitations, primarily due to poor convergence characteristics. This study investigated a design of preconditioners to improve the convergence of iterative methods for this problem.

Index Terms—Condition number, Current and voltage excitations, Eddy current problems, Finite element method.

I. INTRODUCTION

Electric machines design requires an accurate evaluation of eddy current losses in windings driven by high-frequency power sources such as pulse width modulation (PWM). Numerical analysis using the finite element method (FEM) enables accurate determination of eddy current distributions.

However, iterative methods for solving linear systems in eddy current problems with current and voltage excitations [1] have demonstrated poor convergence in some cases. This persistent challenge motivates the present research, which aims to elucidate the underlying convergence degradation mechanisms and develop an efficient preconditioner that improves the convergence of iterative methods while suppressing additional computational overhead.

II. METHOD

A. Governing Equations

Let us consider an eddy current problem with current and voltage excitations where an analysis domain Ω is excited through boundary conditions. The governing equations are

$$\nabla \times \mathbf{H} = \mathbf{J}_0 + \mathbf{J}_{\text{eddy}}, \quad \text{in } \Omega, \quad (1)$$

$$\nabla \cdot (\mathbf{J}_0 + \mathbf{J}_{\text{eddy}}) = 0, \quad \text{in } \Omega_c, \quad (2)$$

where $\mathbf{H}$ and $\mathbf{J}_0, \mathbf{J}_{\text{eddy}}$ are the magnetic field, external current density and eddy currents, respectively. The analysis domain Ω includes the conductive domain Ω_c which has electrode terminals $\Gamma_{\text{in}}, \Gamma_{\text{out}}$ connected to the external circuit. The boundaries consist of Dirichlet and Neumann boundaries Γ_D, Γ_N in addition to the electrode terminals. The boundary conditions are given as follows

$$\mathbf{B} \cdot \mathbf{n} = 0, \quad \mathbf{E} \times \mathbf{n} = \mathbf{0}, \quad \text{on } \Gamma_D \cup \Gamma_{\text{in}} \cup \Gamma_{\text{out}}, \quad (3)$$

$$\mathbf{H} \times \mathbf{n} = \mathbf{0}, \quad \text{on } \Gamma_N \quad (4)$$

where $\mathbf{B}, \mathbf{E}$ and $\mathbf{n}$ are the magnetic flux density, electric field and outward normal unit vector at the boundaries, respectively.

Without loss of generality, we restrict ourselves to the case of single-phase winding excited by an external voltage source. The finite element discretization of the system based on the A-V formulation results in

$$\begin{bmatrix} K_{ee} & K_{ev} \\ K_{ve} & K_{vv} \end{bmatrix} \begin{bmatrix} \mathbf{a} \\ \phi \end{bmatrix} = \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix} U \quad (5)$$

where U is the voltage applied between Γ_{in} and Γ_{out} , and $\mathbf{a}, \phi$ are unknown vectors for the magnetic vector potential and electric scalar potential, respectively.

B. Solution method for linear systems

The system of equations (5) is typically solved using preconditioned conjugate gradient (PCG) method. The incomplete Cholesky (IC) preconditioner [2] is widely used because of its robustness, parallel processing capability, and low computational complexity. However, when considering current and voltage excitations, as in (5), the IC preconditioned CG method (ICCG) has demonstrated poor convergence characteristics in some cases.

Existing approaches to address convergence challenges include multigrid methods [3] and auxiliary space preconditioning (ASP) [4]. However, these techniques have limitations. Multigrid methods, although effective for convergence improvement, face challenges in large-scale parallel computation and often require complex parameter tuning. ASP is designed for curl-curl problems, including eddy current problems, whereas it has been primarily formulated for time-domain problems, with limited reported effectiveness for frequency-domain problems.

The fundamental challenge lies in the computational trade-off between preconditioner strength and computational efficiency. Stronger preconditioners typically improve convergence but increase computational costs and reduce parallel processing capabilities.

C. Design of Preconditioner

This study was motivated by the design of an efficient preconditioner for eddy current problems with current and voltage excitations.

In our preliminary numerical experiment, we observed that the nonzero minimum singular values of K_{ee} and K_{vv} decreased when the winding length increased, which can be explained by eigenvalue analysis of the 1-D Laplace equations. A theoretical analysis of the minimum singular value of the matrix in (5) [5] suggests that strong preconditioning for K_{vv} can improve the condition number of the system. Based on this analysis, we

propose electro-magnetic decoupling (EMD) preconditioners, which separate the preconditioner for the electric scalar potential and magnetic vector potential to optimize the convergence of iterative solvers. The EMD preconditioner is defined as

$$M = \begin{bmatrix} M_1 & O \\ O & M_2 \end{bmatrix} \quad (6)$$

where M_1 is a preconditioning matrix for K_{ee} , and M_2 is a preconditioning matrix for K_{vv} . To design an efficient preconditioner, the second preconditioner is typically chosen to be stronger than the first preconditioner. In this study, “preconditioner A is stronger than preconditioner B” means that the minimum singular value when preconditioner A is applied is larger than when preconditioner B is applied. The multigrid, domain decomposition (DDM) [6], and direct methods can be candidates for the second preconditioner when the IC method is used as the first preconditioner. Note that when using the A-formulation, the EMD preconditioner (6) takes the form of an additive Schwarz preconditioner. The advantages of the EMD preconditioner are as follows:

1. Computational efficiency: Because the size of K_{vv} is smaller than that of K_{ee} , we can suppress the additional computational overhead even if the second preconditioner has a higher computational complexity than that of the first preconditioner.
2. Parallel processing capability: The first and second preconditioners are applied in parallel due to the block diagonal structure. In addition, the parallelization of the first and second preconditioners can be independently extracted.
3. Convergence improvement: The preconditioner drastically improves the convergence of the iterative method for solving eddy current problems with current and voltage excitations compared to the IC preconditioner.
4. Application domain: Both the time- and frequency-domain problems can be solved with the preconditioner.

III. NUMERICAL RESULTS

We consider a 21-turn winding model excited by an external voltage source as shown in Fig. 1, where the numbers of nodes and edges in the mesh are 211,337 and 1,441,102. The conductivity of the winding is $\sigma = 1.0 \times 10^7$ S/m, and the relative permeability of the core is $\mu_r = 1000$. The frequency of the source was fixed at 1 kHz, thus the system was solved in the frequency-domain. The IC preconditioner was used for the first preconditioner, and three preconditioners were tested for the second preconditioner: an algebraic multigrid method (AMG) of V and W cycles, and the two-level additive Schwarz method (ASM) based on the generalized eigenproblems in the overlaps (GenEO) [6]. To validate the proposed method, we compared the results with those obtained using the IC preconditioner.

We solved (5) with PCG. The convergence threshold was 1.0×10^{-8} for the relative residual. The computational resources used for the computation were Intel Core i9-10980XE and DDR4-2933, rank 1, 8GBx8. OpenMP was used to parallelize the iterative method. In ASM-GenEO, the domain Ω_c was partitioned into 18 subdomains, and a coarse space was

constructed in advance. The number of iterations and the computational time are listed in Table 1. We can see that the number of iterations of the EMD preconditioner was reduced to less than one-twentieth, and the computational time was reduced to approximately one-tenth compared to the IC preconditioner. The difference between the reduction rate of the iteration and the speed-up rate is due to the parallel processing capabilities and additional computational overhead of the second preconditioner. Because the parallel processing of AMG is less effective than that of IC, the parallelization efficiency degrades in IC-AMG. In the case of IC-DDM, the additional computational overhead is large because a direct solver is used to solve the subdomain problems. These characteristics have a significant influence on computational time when solving large-scale problems. In the full paper, we report the application of the EMD preconditioner to several large-scale problems and discuss the relation between the computational efficiency and the combination of the different preconditioners against the problem size.

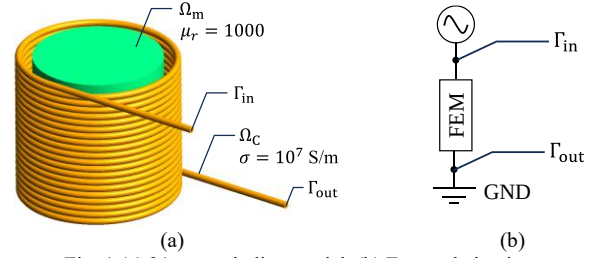


Fig. 1 (a) 21-turn winding model, (b) External circuit

TABLE 1 NUMBER OF ITERATIONS AND COMPUTATIONAL TIME

	Conv.	EMD (proposed)		
Method	IC	IC-AMG (V cycle)	IC-AMG (W cycle)	IC-DDM (ASM-GenEO)
Iteration	8802	364	322	439
Time [s]	397.84	31.78	41.62	45.31

IV. CONCLUSION

In this study, we propose the EMD preconditioners to solve eddy current problems with current and voltage excitations. The proposed method improves the condition number of the system while suppressing the additional computational overhead in iterative methods. In the full paper, the proposed preconditioner is applied to large-scale eddy current problems.

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