

Design Sensitivity Analysis for Shape Optimization of Electromagneto-Quasistatic System in Frequency Domain

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A shape sensitivity analysis is presented for the optimal design of a time-harmonic electromagneto-quasistatic (EMQS) system. The design variable is the interface boundary, and its equation of motion is described using the concept of material derivatives. The shape sensitivity formula is derived from a continuous functional, which represents an augmented objective function that characterizes the EMQS system and its performance through state variables, rather than from discretized differential-algebraic equations. The derived formula is obtained analytically as a surface integral, composed of state and adjoint variables, along with design velocity on the boundary. The deformation of the design variable is computed by combining the design velocity with the level set method. Two numerical examples are presented to demonstrate the feasibility of the proposed sensitivity analysis.

Index Terms—Adjoint Sensitivity Analysis, Continuum Approach, Darwin Model, Electromagneto-quasistatic System, Shape Design.

I. INTRODUCTION

RECENTLY, the electromagneto-quasistatic (EMQS) system, also known as the Darwin model, has gained attention for its ability to simultaneously account for capacitive, resistive, and inductive phenomena in electromagnetic (EM) devices operating at low-to-intermediate frequencies [1], [2]. Previous studies have primarily focused on the numerical implementation of the Darwin model using the finite element method (FEM), with the ultimate goal of computer-aided design (CAD). In CAD, shape optimization is a key area, as the geometric design significantly influences the performance of EM devices. However, to the best of our knowledge, optimization methods for the shape design of the EMQS system have not been developed, in contrast to the established methods in electro- and magneto-quasistatic systems [3], [4].

Continuum sensitivity analysis can be a promising deterministic method for the optimal shape design of the EMQS system. First, it takes advantage of the computational efficiency of the adjoint variable method, which drastically reduces the number of required numerical analyses, compared to the finite difference method [4]. Second, it benefits from the computational accuracy of the continuum approach, where discretization errors do not influence differentiation, unlike in discrete methods [5]. Finally, continuum sensitivity analysis can be implemented using existing FEM code because state and adjoint variables share the same bilinear form.

This study presents a continuum-based shape sensitivity analysis of the EMQS system that a sinusoidal voltage is imposed on its boundary, consistent with previous studies [3], [4]. The shape sensitivity formula is derived and verified through two examples.

II. PROBLEM DEFINITIONS

Let $\Omega \in \mathbb{R}^3$ be the domain, Ω_C be the conductive part, Ω_I be the insulative part, Ω_p be the objective domain, Γ be the outer boundary, γ be the interface boundary, and $\mathbf{n}$ be the outward normal vector.

A. Forward Problem

State equations are expressed as

$$\nabla \times (\nu \nabla \times \mathbf{A}) + j\omega\sigma\mathbf{A} + (\sigma + j\omega\epsilon)\nabla\phi = \mathbf{0} \quad \text{in } \Omega \quad (1)$$

$$-\nabla \cdot (j\omega\sigma\mathbf{A} + (\sigma + j\omega\epsilon)\nabla\phi) = 0 \quad \text{in } \Omega_C \quad (2)$$

$$-\nabla \cdot (j\omega\epsilon\nabla\phi) = 0 \quad \text{in } \Omega_I \quad (3)$$

where ϕ is the state scalar variable, $\mathbf{A}$ is the scalar vector variable ($\nabla \cdot \mathbf{A} = 0$), j is the imaginary unit number, σ is the conductivity, ϵ is the permittivity, and ν is the reluctivity. The matching conditions are expressed as

$$\phi = \phi_0 \quad \text{on } \Gamma_B(\Gamma_E) \quad (4)$$

$$\nabla\phi \cdot \mathbf{n} = 0 \quad \text{on } \Gamma_H(\Gamma_J, \Gamma_D) \quad (5)$$

$$\mathbf{n} \times \mathbf{A} = \mathbf{0} \quad \text{on } \Gamma_B(\Gamma_E) \quad (6)$$

$$\mathbf{n} \times (\nu \nabla \times \mathbf{A}) = \mathbf{0} \quad \text{on } \Gamma_H(\Gamma_J, \Gamma_D) \quad (7)$$

where ϕ_0 is constant. Variational state equation is expressed as

$$a = \int_{\Omega} [\nu \mathbf{B}(\mathbf{A}) \cdot \mathbf{B}(\bar{\mathbf{A}})] d\Omega + \frac{1}{j\omega} \int_{\Omega} [(j\omega\sigma\mathbf{A} + (\sigma + j\omega\epsilon)\nabla\phi) \cdot (j\omega\bar{\mathbf{A}} + \nabla\bar{\phi})] d\Omega \quad (8)$$

where $\mathbf{B}(\cdot)$ is the curl operator of its argument, $\bar{\phi}$ is the scalar test function, and $\bar{\mathbf{A}}$ is the test vector function.

B. Design Problem

An objective function, which describes system performance, is defined as a volume integral as

$$F = \int_{\Omega} [g(\phi, \nabla\phi, \mathbf{A}, \mathbf{B}(\mathbf{A}))] m_p d\Omega \quad (9)$$

where m_p is a characteristic function indicating Ω_p ; it is unity in Ω_p and zero elsewhere. The augmented objective function is defined as follows:

$$G = -a + F, \quad \forall (\bar{\mathbf{A}}, \bar{\phi}) \in (\mathbf{H}_{\Gamma_B}(\mathbf{curl}, \Omega) \times \mathbf{H}_{\Gamma_E}^1(\Omega)) \quad (10)$$

where the test functions play the roles of both the Lagrange multipliers and the adjoint variables.

III. DERIVATION OF SHAPE SENSITIVITY FORMULA

The material derivative of (10) is simply expressed as

$$\dot{G} = -\dot{a} + \dot{F}, \quad (11)$$

where the dot over the letter indicates the material derivative. For adjoint scalar variable λ and adjoint vector variable $\mathbf{\Lambda}$, adjoint equations are defined as

$$\begin{aligned} \nabla \times \nu \mathbf{B}(\mathbf{\Lambda}) - \nabla(\nu \nabla \cdot \mathbf{\Lambda}) + (j\omega\sigma \mathbf{\Lambda} + (\sigma + j\omega\epsilon)\nabla\lambda) \\ = (\mathbf{g}_A + \nabla \times \mathbf{g}_B)m_p, \quad \text{in } \Omega \end{aligned} \quad (12)$$

$$\begin{aligned} -\nabla \cdot (j\omega\sigma \mathbf{\Lambda} + (\sigma + j\omega\epsilon)\nabla\lambda) \\ = j\omega(g_\phi - \nabla \cdot \mathbf{g}_{\nabla\phi})m_p, \quad \text{in } \Omega_C \end{aligned} \quad (13)$$

$$-\nabla \cdot (j\omega\epsilon \nabla\lambda) = j\omega(g_\phi - \nabla \cdot \mathbf{g}_{\nabla\phi})m_p, \quad \text{in } \Omega_I \quad (14)$$

where $g_\phi = [\partial g / \partial \phi]^T$, $\mathbf{g}_{\nabla\phi} = [\partial g / \partial \nabla\phi]^T$, $\mathbf{g}_A = [\partial g / \partial \mathbf{A}]^T$, $\mathbf{g}_B = [\partial g / \partial \mathbf{B}]^T$, and T is the transpose operator. The adjoint variables satisfies the homogeneous matching conditions on the Dirichlet and Neumann boundaries, respectively. The following gauge condition with the gauge fixing is imposed on the adjoint vector variable: $\nabla \cdot \mathbf{\Lambda} = 0$. For γ_{12} between Ω_1 and Ω_2 , the derived sensitivity formula is expressed as

$$\begin{aligned} \dot{G} = & \int_{\gamma_{12}} [\nu_2 - \nu_1] [\mathbf{B}(\mathbf{A}_1)] \cdot [\mathbf{B}(\mathbf{A}_2)] V_{n12} d\Gamma \\ & + \int_{\gamma_{12}} \left[\frac{\sigma_2 - \sigma_1}{j\omega} \right] [\nabla\phi_1 + j\omega\mathbf{A}_1] \cdot [\nabla\lambda_2 + j\omega\mathbf{A}_2] V_{n12} d\Gamma \\ & + \int_{\gamma_{12}} [\epsilon_2 - \epsilon_1] [\nabla\phi_1] \cdot [j\omega\mathbf{A}_2 + \nabla\lambda_2] V_{n12} d\Gamma \\ & + \int_{\gamma_{12}} [(\mathbf{B}(\mathbf{A}_2) - \mathbf{B}(\mathbf{A}_1)) \cdot \mathbf{t}] [\mathbf{g}_{B1} m_p \cdot \mathbf{t}] V_{n12} d\Gamma \\ & + \int_{\gamma_{12}} [\nabla\phi_2 - \nabla\phi_1] \cdot \mathbf{n}_{12} [\mathbf{g}_{\nabla\phi_1} m_p \cdot \mathbf{n}_{12}] V_{n12} d\Gamma \\ & - \int_{\gamma_{12}} [(g_2 - g_1)m_p] V_{n12} d\Gamma, \end{aligned} \quad (15)$$

where $\mathbf{n}_{12}$ is the normal vector directed from Ω_1 to Ω_2 , $\mathbf{t}$ is the tangential vector, $\mathbf{V}$ is the Euler velocity, and V_{n12} is the design velocity defined as $V_{n12} = \mathbf{V} \cdot \mathbf{n}_{12}$.

IV. NUMERICAL EXAMPLES

Problem Definitions: As shown in Fig. 1, the axisymmetric capacitor is tested. The design objective is to obtain a targeted flux density $\mathbf{B}_{\text{target}}$ in Ω_p by deforming γ_{12} . The objective function is defined as

$$F = \frac{1}{2} \int_{\Omega_p} [\nu_0 \|\mathbf{B}(\mathbf{A}) - \mathbf{B}_{\text{target}}\|^2] d\Omega, \quad (16)$$

where $\mathbf{B}_{\text{target}}$ is the numerical solution of the magnetic flux density when γ_{12} is a cylinder with a radius of 1 cm.

Optimization Algorithm: Three steps are repeated until (16) satisfies a stopping criterion, defined as the bounded value of (16) when γ_{12} is a cylinder with a radius of 1 ± 0.001 cm. In the first step, the state equations are solved. Next, (16) is evaluated, and $\mathbf{g}_B$ is specified as a forcing term for the adjoint variables: $\nu_0 [\mathbf{B}(\mathbf{A}) - \mathbf{B}_{\text{target}}]^T$. In the second step, the adjoint equations are solved. The design velocity is determined from (15) to minimize (16). Finally, in the third step, the

level-set equation is solved to compute variations in γ_{12} : $\partial\varphi/\partial\bar{t} + V_n|\nabla\varphi| = 0$. The new geometry is then generated by finding the zeros of φ .

Result: Figure 2 illustrates that the objective function converges to the stopping criterion as the number of iterations increases, with the algorithm halting at 42 iterations. The interface boundary gradually becomes cylindrical (or flat) as shown in Fig. 2. In addition, the objective function converges to the stop criteria as shown in Fig 3.

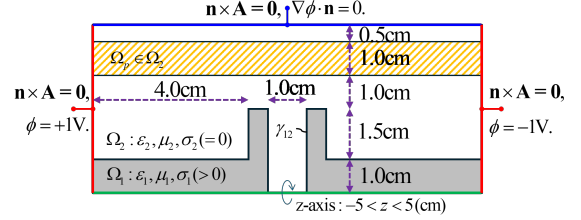


Fig. 1. 2D axisymmetric capacitor: The conductivity of Ω_1 is 10^6 S/m. The relative permittivity and permeability are 1 for Ω_1 and Ω_2 , respectively. The operating frequency is 100 kHz.

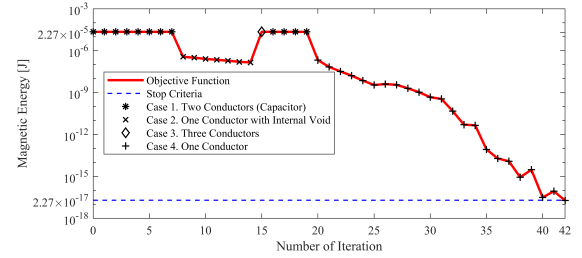


Fig. 2. Evolution of objective function

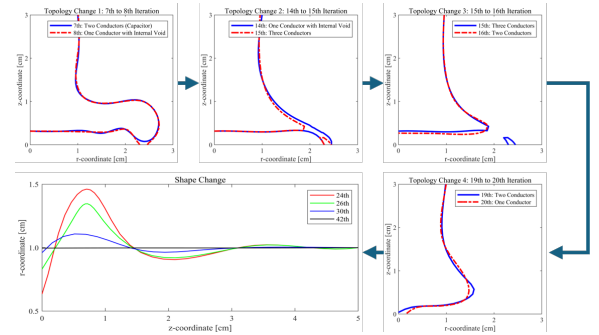


Fig. 3. Evolution of interface boundary

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