

Newton Predictor-Corrector Scheme for the Cauer Circuit Representation of Dynamic Hysteretic Property

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The combination of the play model and the Cauer circuit is an efficient model to represent the dynamic hysteretic properties of magnetic materials. It requires the numerical solution of a nonlinear ordinary differential equation. To avoid the nonlinear iterations, we introduce a new predictor-corrector (PC) scheme called Newton PC scheme to the hysteretic Cauer circuit. We compared the Newton PC scheme with another PC scheme called Heun PC scheme. We found that the Newton PC scheme improves the accuracy successfully even when the Heun PC scheme is not very effective.

Index Terms—Cauer circuit, dynamic hysteresis, error estimation, predictor-corrector scheme.

I. INTRODUCTION

THE increase in the switching frequency of the pulse width modulation (PWM) requires knowing the high-frequency response of magnetic materials to design magnetic devices.

Magnetic materials generally have hysteretic properties. The play model is an efficient method of expressing the static hysteresis property [1]. To represent the dynamic property of magnetic material due to the eddy current, the Cauer circuit is an efficient tool [2]. By combining the play model and the Cauer circuit, the dynamic hysteretic property of magnetic material can be expressed.

The Cauer circuit representation results in a nonlinear ordinary differential equation (ODE). To avoid the possibility that the iterative solution of nonlinear equations may fail due to the hysteresis, the predictor-corrector (PC) scheme has been developed [3]. The benefit of the PC scheme is that errors can be estimated from the difference between the predictor and corrector, which enables a variable time-step scheme according to the estimated errors. Ref. [4] proposed a new PC scheme, which is more accurate than that in [3] in the case of the non-hysteretic motor plant model. This study applies the new PC scheme to the hysteretic Cauer circuit and performs the error analysis of the two PC schemes.

II. PREDICTOR-CORRECTOR SCHEME

The Cauer circuit is shown in Fig. 1, where $\Phi_1, \Phi_3, \Phi_5, \dots$ are the magnetic fluxes, $i_1, i_3, i_5, \dots$ are the currents, and $R_2, R_4, \dots$ are the resistances, representing the eddy current loss.

The ODE of the Cauer circuit is given as

$$\frac{d\Phi}{dt} = e(t) - r i(\Phi), \quad (1)$$

where

$$e(t) = U^T V_{in}, \quad r = U^T R U, \quad (2)$$

$$\Phi = (\Phi_1, \Phi_3, \Phi_5, \dots)^T, \quad i = (i_1, i_3, i_5, \dots)^T, \quad (3)$$

$$V_{in} = (v_s, 0, 0, \dots)^T, \quad R = \text{diag}(0, R_2, R_4, \dots), \quad (4)$$

U is the upper triangular matrix whose elements are all 1.

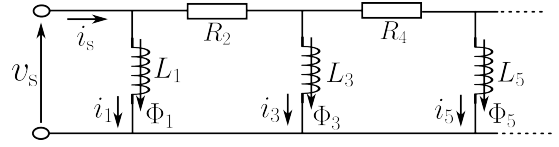


Fig. 1. Cauer circuit.

When solving (1) by the back Euler (BE) scheme, the arising nonlinear equations are often solved by Newton's method. To avoid Newton iterations, we use the semi-implicit method as

$$\begin{aligned} \Phi^{k+1} &= \Phi^k + \Delta t \{ e(t^{k+1}) \\ &\quad - r[i(\Phi^k) + \frac{\partial i(\Phi^k)}{\partial \Phi}(\Phi^{k+1} - \Phi^k)] \}, \end{aligned} \quad (5)$$

where k is the time index, and Δt is the time step. Equation (5) is a linear equation for Φ^{k+1} .

The semi-implicit method above may not be very accurate. To improve accuracy, the PC scheme can be used. In the PC scheme, Φ^{k+1} in the semi-implicit method (5) is used as a predictor Φ^* . After obtaining the predictor, the accuracy can be improved using two different correctors as follows.

Ref. [3] corrected the predictor Φ^* to obtain more accurate Φ^{k+1} as

$$\begin{aligned} \Phi^{k+1} &= \Phi^k + \Delta t \{ e(t^{k+1}) \\ &\quad - r[i(\Phi^k) + N^*(\Phi^{k+1} - \Phi^k)] \}, \end{aligned} \quad (6)$$

where

$$N^* = \frac{\frac{\partial i(\Phi^*)}{\partial \Phi} + \frac{\partial i(\Phi^k)}{\partial \Phi}}{2}. \quad (7)$$

We refer to the PC scheme with (6) as Heun PC scheme because the correction scheme is similar to that of Heun's method.

Ref. [4] gives another corrector as

$$\begin{aligned} \Phi^{k+1} &= \Phi^* + \Delta t \{ e(t^{k+1}) \\ &\quad - r[i(\Phi^*) + \frac{\partial i(\Phi^*)}{\partial \Phi}(\Phi^{k+1} - \Phi^*)] \}. \end{aligned} \quad (8)$$

We refer to the PC scheme with (8) as Newton PC scheme because this corrector is based on Newton's method.

III. NUMERICAL RESULT

A sheet magnetic material is analyzed. The thickness of the sheet is 3×10^{-4} m, and the conductivity is 3.933×10^6 S/m. The number of ladder stages in the Cauer circuit is 4. The number of time steps per cycle is 100. The number of periods is 2.

In the Cauer circuit, the first inductor represents the static hysteretic property whereas the latter stage inductors handle the dynamic property. There are several approximations for the latter inductors to include dynamic hysteresis. In this study, we used the simple approximation [5] and the reversible approximation [6]. In the simple approximation [5], the current i_{2m-1} of the inductor L_{2m-1} ($m = 2, 3 \dots$) is given as

$$i_{2m-1}(\Phi_{2m-1}) = H_{\text{static}}((4m-3)\Phi_{2m-1}), \quad (9)$$

where H_{static} is the static hysteretic function represented by the play model and $i_1 = H_{\text{static}}(\Phi_1)$. In the reversible approximation [6], i_{2m-1} is given as

$$i_{2m-1}(\Phi_{2m-1}) = (4m-3)\nu_{\text{rev}}\Phi_{2m-1}, \quad (10)$$

where ν_{rev} is the differential reluctivity of reversible component of $H_{\text{static}}(\Phi_1)$.

We use the three differences below for evaluating the Heun PC scheme and the Newton PC scheme.

$$d_c = \frac{1}{\Phi_{\text{NR}}} \sqrt{\frac{1}{n} \sum_{k=1}^n \|\Phi^{k+1} - \Phi_{\text{NR}}^{k+1}\|^2}, \quad (11)$$

$$d_p = \frac{1}{\Phi_{\text{NR}}} \sqrt{\frac{1}{n} \sum_{k=1}^n \|\Phi^* - \Phi_{\text{NR}}^{k+1}\|^2}, \quad (12)$$

$$d_{p-c} = \frac{1}{\Phi_{\text{NR}}} \sqrt{\frac{1}{n} \sum_{k=1}^n \|\Phi^* - \Phi^{k+1}\|^2}, \quad (13)$$

where k is the time index, Φ_{NR}^{k+1} is the magnetic flux obtained from the BE scheme solved by Newton's method, Φ_{NR} is the maximum value of $\|\Phi_{\text{NR}}^{k+1}\|$ ($k = 1, 2, 3 \dots$), n is the number of the whole steps (200). d_c represents the difference between the corrector and Newton's method. d_p represents the difference between the predictor and Newton's method. d_{p-c} represents the difference between the corrector and the predictor. When d_c is small, it means that the correction by a PC scheme is effective. When d_p and d_{p-c} are close, we can evaluate errors using d_{p-c} without calculating d_p .

Figs. 2(a)(b) show the three differences versus frequency with Heun and Newton PC schemes using the simple approximation. Figs. 3(a)(b) show the differences using the reversible approximation.

Compared with Fig. 2(a), d_c is smaller in Fig. 2(b). In Fig. 2(b), d_p and d_{p-c} are almost the same, which suggests that the Newton PC scheme has a good error correction ability. In Fig. 3(a), the values of d_p and d_c are almost the same, which means that the Heun PC scheme has only a small effect. In contrast, in Fig. 3(b), d_p and d_{p-c} are almost the same and d_c is very small showing that the Newton PC scheme works effectively.

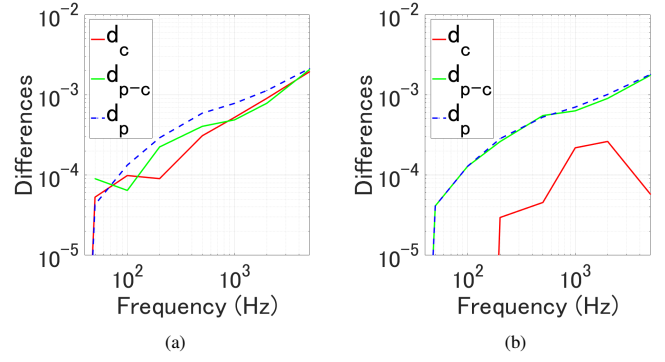


Fig. 2. Differences with the simple approximation: (a) Heun corrector and (b) Newton corrector.

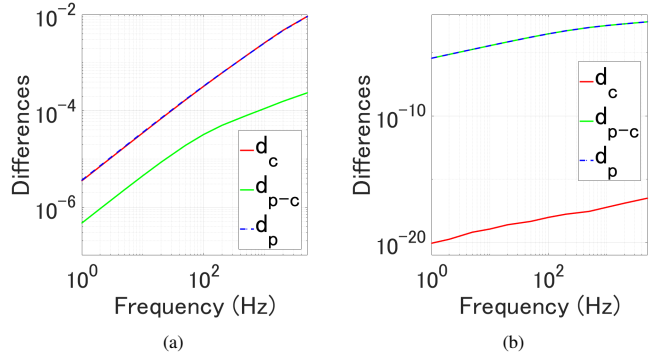


Fig. 3. Differences with the reversible approximation: (a) Heun corrector and (b) Newton corrector.

In this study, we found that the Newton PC is effective in reducing the error and evaluating the error. This leads to a variable time-step PC scheme using the error estimation, which will be discussed in the full paper.

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