

Modeling of Superconducting Wire Carrying Transport Current

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This paper deals with numerical modeling of current density and magnetic field in a superconducting wire carrying AC transport current. For that, we use the finite element method. We observe the current density distribution and including magnetic field distribution in order to study the electromagnetic behavior in superconducting materials. Then we compare the results obtained by using different numbers of filaments in the superconducting wire.

Index Terms—Current density, Electromagnetic Modeling, Finite element analysis, Superconducting filaments.

I. INTRODUCTION

THE USE of High-Temperature Superconductors (HTS) for Alternating Current (AC) transport is currently a reality and the evaluation of the self-field losses is necessary. Although these losses are low, they must be evaluated with precision. This is particularly important because they occur at low temperatures, making the evaluation costly. Classically, the estimation of losses is carried out using formulas derived from Bean's model, developed to characterize Low-Temperature Superconductors (LTS), which are defined by the electric field-current density $E(J)$ characteristic [1]. But for transporting the alternating current at high critical temperature, the real $E(J)$ characteristic differs rather clearly from that of Bean's model. A more realistic model is obtained by a progressive function called the extended Bean's model [2], [3].

A model based on the generalization of Bean's model has been developed at LGEP (now called GeePs) in France. This model has been integrated into our finite element program designed for cartesian and axisymmetric geometries. Although the program is generally used to model the superconducting materials at low critical temperature, it can also be applied to model at high critical temperature. This program calculates the electromagnetic field of a superconducting device using the finite element method (FEM).

II. PROBLEM

The electromagnetic equilibrium is governed by Maxwell's equations relating to the electric field (E), the current density (J), the magnetic field (H), and the magnetic induction (B):

$$\nabla \times \vec{E} = -\partial_t \vec{B}, \nabla \times \vec{H} = \vec{J}, \nabla \cdot \vec{E} = 0, \nabla \cdot \vec{H} = 0 \quad (1)$$

The behavior of a superconducting material is prescribed by its constitutive relations. For B and H , we have $B = \mu_0 H$ where μ_0 is the permeability of the air. For E and J , we use the extended Bean's model as in [2]. In this model, we suppose that the electric field and the current have only one component following the axis of the connection. The scalar equation to solve is written:

$$\partial_t J = \nabla \cdot (\mu_0^{-1} \nabla E) = \mu_0^{-1} \nabla^2 E \quad (2)$$

It is solved by using the finite element method in space and Crank-Nicolson method in time. With each step of time, a convex functional is minimized by an iterative procedure of nonlinear Gauss-Seidel [4]-[6].

We propose to model a long superconducting wire carrying a non-zero total transport current. This transport current, directed along the axis of the superconductor, will primarily generate a magnetic field that varies with time and space. For a given transport current, we consider the modeling of superconducting wire in the following cases. We model a superconducting wire containing 1 at the center, 6 in the first layer, and 12 filaments in the second layer, each filament having a radius of 3.5 mm and an infinite length along the z-axis [7]. For this, we allow a time-varying current $I(t)$ to flow through these superconducting filaments. Additionally, the distance R from the origin to the point where the magnetic field is calculated is known. Given $I(t)$ and R , the magnetic field at the fictitious boundary of the modeled domain could be determined using the following equation:

$$\vec{H} = \frac{I(t)}{2\pi R} \hat{u} \quad (3)$$

where $\hat{u}$ denotes the orthoradial unit vector. And by using the following equation, we obtain the boundary conditions, which in this specific case are neither uniform nor constant.

$$\partial_n E = \mu_0 \partial_t H_{tg} \quad (4)$$

where H_{tg} denotes the tangential component of the applied magnetic field.

III. RESULTS

We applied a sinusoidal current with a frequency of 50 Hz (a period of 20 ms) and a maximum amplitude of 150 A ($I_{max} < I_c$ where $I_c = 177$ A and $J_c = 4,600,000$ A/m²) as shown in Fig. 1. We analyzed the distributions of current density and magnetic field within the study domain at various time points $t_A = 6.25$

ms and $t_B = 11.25$ ms which are defined in Fig. 1. To determine and understand these distributions, we used Bean's model and Maxwell's equations.

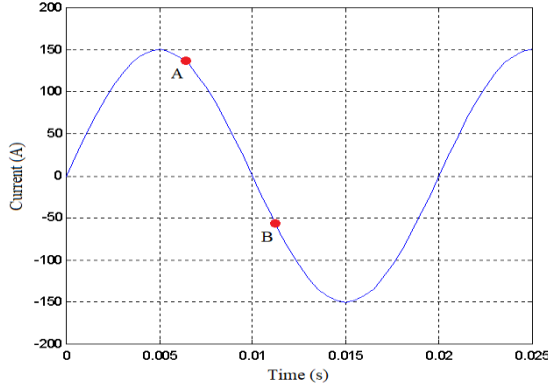


Fig. 1. Total transport current and definition of time points t_A and t_B .

Figures 2, 3 and 4 show the distributions of current density (left) and magnetic field (right) at different time points t_A and t_B in 2D view. We clearly observe the penetration of the magnetic field into the superconducting filaments for all cases. This penetration progresses with a curl whose magnitude is equal to J_c . According to Lenz's law, currents opposing the variations in the magnetic field develop from the outer boundary of the superconducting filaments. We observe that the currents begin to flow from the filament's outer boundary.

According to Bean's model, these currents have a density equal to J_c . This results in a phenomenon similar to the skin effect, with the difference that the current density remains constant over the depth of penetration. Furthermore, the distribution of current density demonstrates symmetry about the center of the filament.

IV. CONCLUSION

The use of superconductors shows now the real applications in the electrical area such as the electrical power cable or the electrical machine. So, we have to seek to model these materials in order to be able to study and analyze all the aspects of them.

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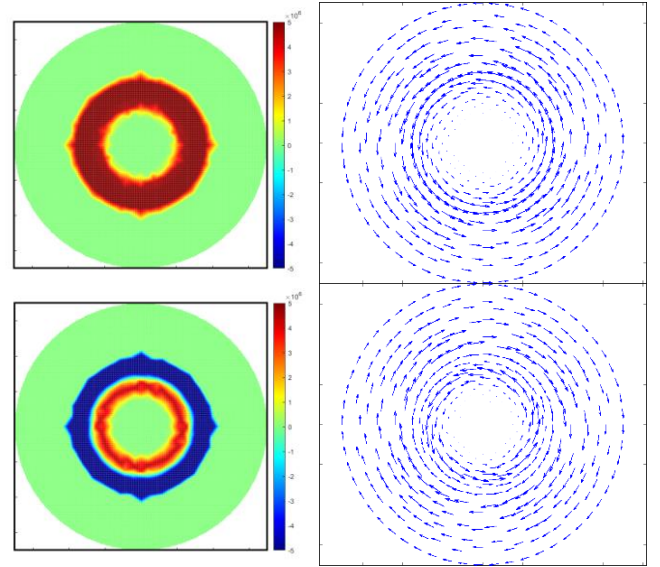


Fig. 2. Distributions of current density (left) and magnetic field (right) at different time points t_A and t_B for one filament at the center of wire.

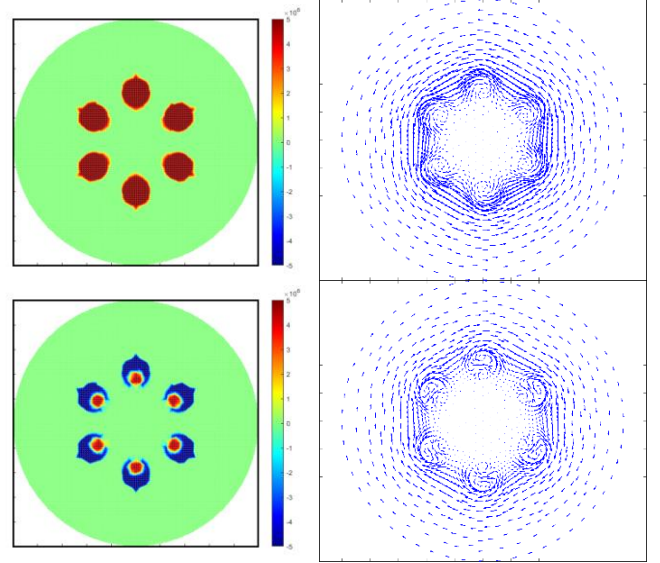


Fig. 3. Distributions of current density (left) and magnetic field (right) at different time points t_A and t_B for 6 filaments in the first layer of wire.

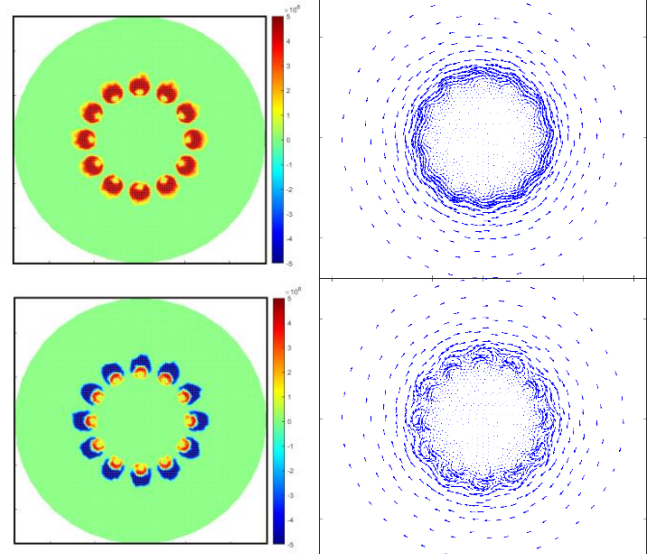


Fig. 4. Distributions of current density (left) and magnetic field (right) at different time points t_A and t_B for 12 filaments in the second layer of wire.