

Bi-Dimensional Inductance Calculations for Unidirectional Carbon Fiber Composite Materials

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This paper presents calculations of self and mutual inductances using an integral approach between long carbon fibers in unidirectional composites materials. The random distribution of fibers is considered via an algorithm designed specifically for this purpose. The algorithm takes into account repulsion constraints and ensures no overlap between the fibers. Simulation results, presented as images, depict the random distributions of fibers with variable volume fractions (from 40% to 70%). The final calculations reveal mutual inductances in the actual state of the fibers and provide insights into values at increasing distances. The magnitude of the inductances, along with resistance calculations included in the extended format of the paper, enables a better understanding of the effects of frequency on these types of materials.

Index Terms—Carbon Fiber Composites, Representative Volume Element, Inductance.

I. INTRODUCTION

In recent years, carbon fiber composites (CFCs) have invaded various industries due to their exceptional properties, including high specific strength and stiffness [1]. These materials are widely used in high-speed vehicles, aircraft [2] and wind turbine blades. Depending on their topologies, they can play important roles, including lightning protection for aerospace systems and electromagnetic shielding [3]. For the detection and verification of defects in these composite materials, several techniques can be applied, such as damage detection by electrical potential monitoring, ultrasonic and eddy current non-destructive testing (ECNDT) [4], hence the constant need to know the electrical, thermal and mechanical properties and frequencies effects. Much of the previous researches were focused on the macroscopic scale behavior of (CFCs) but microscopic scale studies remain essential to advance their applications [5]. In this work, we are interested in the electrical parameters of carbon fibers and more particularly in the interactions induced at the microscopic scale. Many works have addressed the electrical resistances for more or less complex structures presented by two-dimensional and sometimes three-dimensional models. It is always mentioned especially in recent papers the need to study the inductive effects especially with respect to the increase in frequency either desired as the (ECNDT) or the existence of this type of materials in systems containing embedded electronics. For this reason, we start in this work to implement the bi-dimensional inductance calculation for randomly distributed long fibers respecting realistic topologies.

II. ALGORITHM DESCRIPTION

In the domain of generating random distributions, various algorithms are employed, especially the Monte Carlo method being one of most widely used for generating Representative Volume Elements (RVE). However, its packing efficiency is limited, achieving values between 55% and 67.43%. Other methods include cell-cutting techniques, algorithms based on event-driven molecular dynamics theory,

as well as the hard-core model and random shaking model.

These approaches offer diverse strategies to improve the packing density and realism of generated (RVEs).

To achieve this, we developed a method to generate a random distribution of fibers at the micro-scale, adhering to general conditions recommended by Trias in [6] and adopted by other researchers. Three fiber volume fractions were selected for analysis: 40%, 50%, and 60%. The input variables for the algorithm include a fiber radius (r_w)=2.6 μ m, the minimum distance between adjacent fibers (l_{min})= 0.182 μ m (0.07 of radius fiber), and the ratio of the RVE microstructure side length to fiber radius $\delta = 50$. The algorithm combines a hardcore model to adjust fiber positions and an elasticity model to achieve the desired volume fractions. This hybrid approach ensures realistic and efficient random fiber distributions Fig.1.

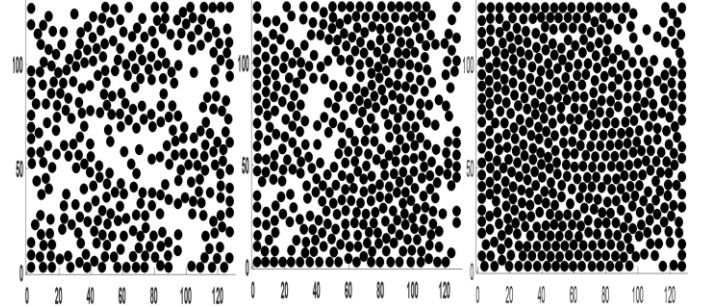


Fig. 1. Results of RVE volume fraction: a) 40% b) 50% c) 60%.

III. FORMULAS FOR CALCULATE PARAMETERS

This study aims to present a two-dimensional model that accurately simulates the random distribution of these unidirectional fibers with a precise and adequate calculation of the self and mutual inductances at their scales. The laws and formulas used are those of the best-known old works such as those of Gover [7] grouped by CR Paul [8]. The basic calculation model is shown in Fig. 2

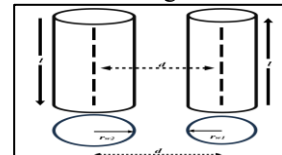


Fig. 2. Elementary model for two parallel equal conductors.

A. Self-Inductance

1) Self-Inductance by Grover

Grover defines the self-inductance by the formula :

$$L = 0.002l \left[\ln \frac{2l}{r_w} - \frac{3}{4} \right] \quad (1)$$

2) Self-Inductance by CR Paul

Paul gives the formula at low frequency, same as Grover's one (Equation (1)), but at high frequency, the adopted expression is:

$$L_p = \frac{\mu_0}{2\pi} l \left[\ln \left(\frac{2l}{r_w} \right) - 1 \right] \quad (2)$$

B. Mutual Inductance

1) Mutual Inductance by Grover

Gover describes mutual inductance in the context of two equal parallel conductors. He explains that the mutual inductance is equivalent to that of filaments passing through the centers of their cross-sections, even when the conductors are nearly in contact. The relationship can be expressed using the length (l) of the conductors and the distance (d) between them, providing an effective approximation for mutual inductance in tightly spaced configurations

$$M_p = \frac{\mu_0}{2\pi} l \left[\ln \left(\frac{1}{d} + \sqrt{1 + \frac{l^2}{d^2}} \right) - \sqrt{1 + \frac{d^2}{l^2}} + \frac{d}{l} \right] \quad (3)$$

2) Mutual Inductance by CR Paul

Mutual inductance between two parallel wires of equal length (l), with currents uniformly distributed over their cross-sections, can be effectively modeled using geometric approximations. In [8], CR Paul describes an approach where the wires are replaced by filaments located along their axes, simplifying the calculation. The mutual partial inductance is then expressed in terms of the geometric mean distance (GMD), defined as the distance between the centers (s) of the circular cross-sections of the wires. This method provides a practical approximation for mutual inductance in electrical systems:

$$\ln D = \ln s \quad (4)$$

Mutual partial inductance

$$M_p = \frac{\mu_0}{2\pi} l \left[\ln \left(\frac{1}{d} + \sqrt{1 + \frac{l^2}{d^2}} \right) - \sqrt{1 + \frac{d^2}{l^2}} + \frac{d}{l} \right] \quad (5)$$

$$M_p \approx \frac{\mu_0}{2\pi} l \left[\ln \left(\frac{2l}{d} \right) - 1 \right]$$

IV. RESULT

The results of this work are derived by adhering to the algorithm's conditions and analyzing the outcomes of the random fiber distributions. Specifically, the distance (d) between fibers generated by the algorithm and the fiber radius (r_w) from the predefined conditions were evaluated. Using the simplifications proposed by Paul, the calculations yielded the following results for fiber volume fraction 60%:

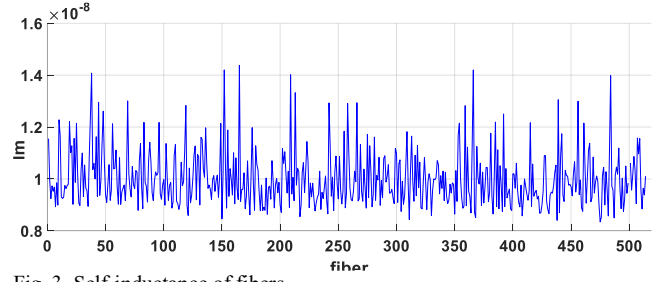


Fig. 3. Self-inductance of fibers.

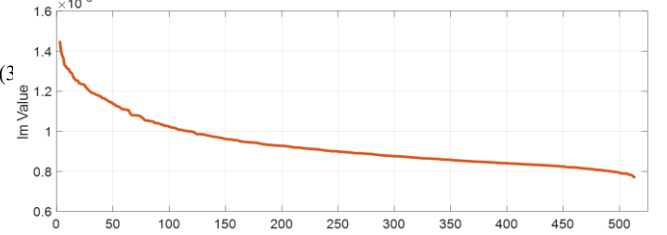


Fig. 4. Sorted Mutual inductance of fiber to others.

V. CONCLUSION

This paper presents a new approach to consider and calculate inductive effects on long carbon fiber composites. The calculation is inspired by mathematical developments of interactions between ordinary conductors in electromagnetism.

The proposed algorithm allows to generate a 2D RVE model of random distributions of unidirectional fibers in composite materials at the microscopic scale. We could not present much, but the adopted algorithm will be able to adapt to several scenarios, including constant and random radii fibers, as well as variable volume fractions up to 70%. The objective of the inductance calculations lies in the knowledge of the importance of the induced phenomena especially during high frequency applications with also the consideration of the resistive effects that will be inserted in the extended part of this work. This way of calculation easily leads to circuit approaches by the generation of full matrices for the resolution of the distributions of electric currents for the parts at the microscopic scale and the physical considerations that go with it. The calculation part of electrical equation systems will be presented also in the extended version of this paper.

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Dear Reviewers,

I would like to sincerely thank you for your valuable time and thoughtful feedback on our abstract entitled " ***Bi-Dimensional Inductance Calculations for Unidirectional Carbon Fiber Composite Materials*** " Your comments and suggestions were highly constructive and greatly contributed to improving the quality and clarity of the work.

I truly appreciate your efforts and expertise, and I am grateful for the opportunity to benefit from your review.

Here some responses to perfect the last version of abstract

Review 1

Contribution of the Submission

In this paper, a self and mutual inductances of carbon fibers taking account of those random distribution are calculated by combining the random distribution modeling and inductance calculation methods. Each modelling and method are not new and the demonstrated calculation example is poor, but the inductance calculation of carbon fibers is interesting.

Comments for the Authors

In this paper, a self and mutual inductances of carbon fibers taking account of those random distribution are calculated by combining the random distribution modeling and inductance calculation methods. Each modelling and method are not new and the demonstrated calculation example is poor, but the inductance calculation of carbon fibers is interesting. In the full paper, more practical calculation examples, such as the frequency characteristics of impedance, the effect of volume fraction of the impedance, etc. should be demonstrated and verified by the experiments.

The other comments on this digest are shown below:

1. The results in Figs. 4 and 5 should be explained in the main text.
 - In full paper
2. What is the difference between "Mutual inductance between one fiber and others" and "Mutual inductance for one fiber and all others" in the labels of Figs. 4 and 5, respectively?
 - It is just problem in translate
3. Please format carefully. For example, "I. INTRODUCTION" should be centering. The first word in "I. INTRODUCTION" should be all caps. The beginning of paragraph should be indented.
 - Yes I will
4. (3) at the right of the equation number (2), and (4) at the right of (3) should be deleted.
 - Yes

Review 2

Contribution of the Submission

The paper derives an approach to describe 2D microstructures on unidirectional composite materials. The approach is then used to calculate, from analytical formulas, the self-inductance and mutual inductance between fibers in the microstructure. Such calculations can be useful when the material is subjected to high frequency electromagnetic field, for instance in the context of eddy current testing.

Comments for the Authors

The paper addresses a topic of interest. I have a few questions.

1. In a real microstructure, all fibers are not rigorously parallel, leading to ellipses rather than circles in 2D. How could that be taken into account? How much would that affect the results?
 - The main idea is the fiber is circles in 2D and there is a small difference in the radius of them in this work we aim to propose the fibers are the same and we have other modelling of different radius and the calculation of them the difference will show in self-inductance but the Mutual is the same as this result.
2. In the generation of the microstructure, the authors set the minimum distance between fibers as an input. How is the value chosen? In real composites, is it not possible to have two fibers in contact?
 - The minimum distance between fibers is input here because we work in 2D for the unidirectional so if there is a contact between two fibers that leads us to contact from the beginning to the end of the fibers and this makes the two fibers just one fiber with a new form a two contact circle.
 - The contact is a real but just as the points or small area of fibers
3. Also a ratio between the RVE microstructure side length and the fiber radius is imposed. How is this value determined?
 - This input from other articles and we have a small analysis of SEM for determining the Geometry of fiber but is not well yet.
 - The ratio between the RVE and composite is important for the static study of the microstructure of composite. In reference [6] this work.
4. Is a periodicity of the RVE imposed? If yes, this should be mentioned. If not, a fiber near the border has less neighbor than a fiber in the middle of the RVE. Isn't that an issue?
 - This study of just one RVE and this for the micro-scale in the macro-scale there is

As I understand it, Fig. 4 and Fig.5 were calculated for a single microstructure. For which volume fraction? Is there no statistical effects that should be studied since the calculation was performed on a single realization of the microstructure?

5. It is not clear how the authors intend to verify and validate their approach? Is there a reference configuration with a known solution to compare with? Or are experimental results available? Maybe a full field approach, such as the finite element method, with periodic boundary conditions could be used to validate the calculations?
 - This is a new idea for study in micro-scale and. And we try to develop it

Review 3

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Contribution of the Submission

In this paper the authors calculate self and mutual inductances in unidirectional composites with long carbon fibers using an integral approach. A custom algorithm ensures realistic, non-overlapping fiber distributions for volume fractions of 40% to 70%. Results reveal mutual inductances based on fiber arrangements and distance, with extended data offering insights into frequency effects.

Comments for the Authors

1. In my opinion, the algorithm is not well described
 - In two pages we try to give just the result in Full paper we will describe the algorithm