

Coenergy-Based Methodology for Modeling Anisotropy in Grain-Oriented Electrical Steel

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This work focuses on the vectorization of the magnetic anisotropy model for grain-oriented steel sheets by exploiting coenergy computation and interpolation methods. The proposed approach is based on a modified representation of coenergy isocontours in elliptical form and introduces further improvements with the integral form of the error functions and Fourier series expansion. This will provide a remedy for the deficiencies of the aforementioned approaches and introduce a more robust framework for modeling and simulating the anisotropic magnetic response.

Index Terms—Grain-oriented electrical steel, Magnetic anisotropy, coenergy modeling, elliptical contours, vectorization.

I. INTRODUCTION

GRAIN-ORIENTED electrical steels (GOES) are crucial materials in energy-efficient electrical machines, where magnetic anisotropy is a critical performance factor [1]. This type of steel sheet is manufactured to exhibit a unique crystal structure in which its grains are oriented toward one direction, i.e., the Rolling Direction (RD). Understanding and accurately modeling the anisotropy is important to improve such machine designs and operations [2]. Traditional scalar magnetic models that have been applied so far turned out to be quite effective for applications but lacked the capability of capturing the vectorial behavior of magnetic fields, particularly when energy-based analyses are involved. The coenergy approach creates a promising framework for enhancing the precision and applicability of such models [3]. Advances in computational techniques have further enhanced the applicability of coenergy-based models, enabling their integration into complex CAD tools for electromagnetic design.

For a long time, modeling such anisotropy has been a challenge. The early approaches used diagonal permeability tensors to define the permeability along major directions such as RD, Transverse Direction (TD), and Orthogonal Direction (OD), assuming linear interpolation for the other directions, which proved inaccurate [4]. The nonlinear elliptical models were improvements but still did not capture the full character of the complex anisotropic behavior [5]. The later models based on the first magnetization curves had improved accuracy but required extensive experimental data [6]. The orientation distribution theory models tried to minimize this dependency, but the need for many nonlinear coefficients in those models made them unsuitable for practical engineering use. Recent works, inspired by [7], model the $B(H)$ relationship using Gaussian cumulative distribution functions; this approach has proved promising in balancing accuracy and practicality [8]. However, it is still a scalar model and hence cannot fully capture the full vectorial behavior of $\mathbf{B}$ and $\mathbf{H}$. This motivates the development of methods that extend these models into a

vectorial framework, to fully represent anisotropic magnetic behavior.

In most practical situations, the magnetization curves along the RD and TD are known. Still, it is very important to determine the behavior of magnetic fields in the non-principal directions because $\mathbf{B}$ and $\mathbf{H}$ are not always parallel. Early computational models, such as those based on stored coenergy or energy density functions [9], provided foundational methods to represent anisotropic $B(H)$ curves. Coenergy-based models have emerged, effectively describing the vectorial $\mathbf{B}(\mathbf{H})$ relationship and capturing key features like decoupled magnetization at low fields and the difficult direction at higher fields [3]. By taking advantage of the fact that the relationship $B(H)$ is equivalent to the coenergy density $\omega'(H)$, it provides an easy representation of anisotropy, but still physical. The coenergy density $\omega'(H)$ represents the volumetric electromagnetic coenergy stored in the material as a scalar function of the applied field H :

$$\omega'(\mathbf{H}) = \int_0^{\mathbf{H}} \mathbf{B}(\mathbf{H}) d\mathbf{H}. \quad (1)$$

The $\mathbf{B}(\mathbf{H})$ relationship is derived directly as the gradient of $\omega'(\mathbf{H})$, ensuring consistency with energy conservation principles:

$$\mathbf{B}(\mathbf{H}) = \frac{\partial \omega'}{\partial \mathbf{H}}. \quad (2)$$

The coenergy method simplifies the modeling of magnetic anisotropy by reducing the degree of complexity in the representation of the material's behavior. Instead of a vectorial relationship $\mathbf{B}(\mathbf{H})$, this method uses contours of equal coenergy density $\omega'(\mathbf{H})$, naturally encoding the angular dependence of the magnetic response. Such simplification reduces the level of interpolating and parameterizing experimental data.

This study aims to vectorize the magnetic anisotropy model of grain-oriented electrical steel sheets using a coenergy-based formulation. The approach begins with computing the magnetic polarization $\mathbf{J} = \mathbf{B} - \mu_0 \mathbf{H}$ from measured data and fitting its directional components using a Langevin-based function

model. This smooth, physically consistent model facilitates accurate computation of the stored coenergy ω' via integration. The coenergy is then interpolated and transformed into polar coordinates $(|H|, \phi_H)$ to define anisotropy through elliptical iso-contours, from which directional parameters are extracted. Finally, the magnetic flux density components (B_x, B_y) are reconstructed by differentiating the coenergy with respect to (H_x, H_y) . This method offers a fully vectorial description of magnetic anisotropy, overcoming the limitations of previous scalar models and enabling accurate modeling in arbitrary in-plane directions.

II. METHODOLOGY

- 1) Compute the magnetic polarization components (J_x, J_y) using the relation $\mathbf{J} = \mathbf{B} - \mu_0 \mathbf{H}$, by using measured magnetic field and magnetic flux density in the RD and TD, and derive the polar coordinates $(|H|, \varphi_H)$ and $(|J|, \varphi_J)$.
- 2) Fit the experimental polarization curves along the RD, TD, and an additional direction (any third direction ATD) using a composite Langevin model to obtain $J_{RD}(H)$, $J_{TD}(H)$, and $J_{ATD}(H)$ (see Fig. 1).

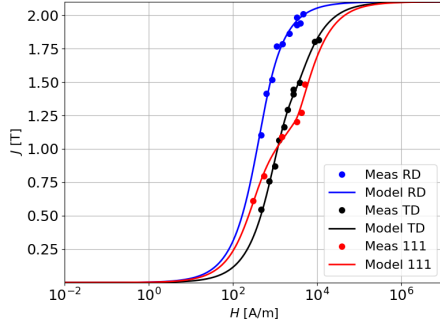


Fig. 1. Experimental and Fitted Polarization Curves Along the RD, TD, and (111) Direction. The third direction here corresponds to (111), an example of an additional in-plane direction.

- 3) Compute the stored coenergy functions $\omega'_{RD}(H)$, $\omega'_{TD}(H)$, and $\omega'_{ATD}(H)$ by integrating $J(H)$ over H (see Fig. 2).

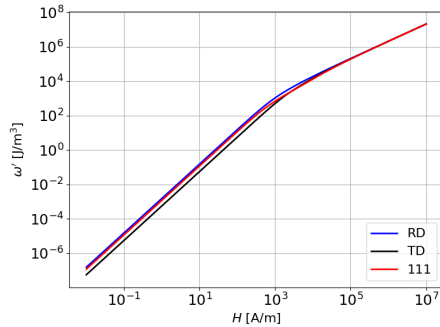


Fig. 2. Coenergy curves computed along the rolling direction (RD), transverse direction (TD), and (111) direction), based on the fitted Langevin function model.

- 4) Define iso-coenergy contour levels ω'_0 within the experimental range and determine the corresponding fields H_{x0} and H_{y0} using the fitted functions.
- 5) Determine the anisotropy factor $n(\omega'_0)$ at each coenergy level by solving the elliptical contour equation:

$$\left(\frac{H_x}{H_{x0}}\right)^n + \left(\frac{H_y}{H_{y0}}\right)^n = 1$$

based on the field values along RD, TD, and any additional direction.

- 6) Differentiate numerically the stored coenergy $\omega'(H_x, H_y)$ with respect to H_x and H_y to compute the magnetic flux density components:

$$B_x = \frac{\partial \omega'}{\partial H_x} + \mu_0 H_x, \quad B_y = \frac{\partial \omega'}{\partial H_y} + \mu_0 H_y$$

thus, reconstructing the vectorial magnetic response from the coenergy formulation.

III. CONCLUSION

The paper presents a general method for vectorizing the GOES magnetic anisotropy model through coenergy analysis. The approach builds upon experimental data to compute coenergy functions and define anisotropy parameters, enabling the development of a robust framework for vectorial magnetic modeling. While this work lays the foundation, the full paper will detail the coenergy modeling, inspired by [6], to further refine the representation of anisotropic behavior. This methodology marks a significant step toward more accurate analysis and optimization of electrical machines by overcoming the limitations of scalar models.

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