

Time-Domain Modeling of High-Frequency Soft Magnetic Materials Based on Cauer Networks

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We propose a novel approach for modeling soft magnetic materials (SMMs) at high frequency under both dielectric and magnetic excitations in time-domain. Cauer networks (CNs) are first synthesized from analytical material models obtained by homogenization for modeling the effective electromagnetic behavior of SMMs in both frequency- and time-domains. The network components depend explicitly on the physical and geometric properties of the material microstructure. The CNs are then coupled with finite element (FE) models to simulate the macroscopic behavior of SMMs in both frequency- and time-domains. Finally, numerical simulations are validated against measurements. Mn-Zn ferrite and soft magnetic composite samples are investigated. The proposed approach provides a practical framework for efficiently modeling SMMs under different excitations in both frequency- and time-domains.

Index Terms—Cauer network, complex permeability, complex resistivity, ferrites, finite element method, homogenization, soft magnetic composite, time-domain.

I. INTRODUCTION

SOFT MAGNETIC MATERIALS (SMMs), e.g. soft ferrites and soft magnetic composites (SMCs) are gaining popularity in high-frequency power electronics applications such as DC-DC converters and high frequency-transformers [1], [2]. Datasheets of such materials provide characteristics only in the frequency-domain, but power electronics engineers are more interested in how the material behaves in the time-domain under nonsinusoidal excitations. Furthermore, SMMs have a heterogeneous microstructure that greatly affects their electromagnetic behavior. Modeling such materials while considering their complex microstructure is not a simple task, especially in the time-domain. Cauer networks (CNs) and Foster networks are widely used for modeling SMMs [3]–[5] as they provide immediate information about the material behavior in both frequency- and time-domains. Sato and Igrashi presented a homogenization method for time-domain analysis of the macroscopic complex permeability of SMC [3]. Liu et al. proposed a Foster network for modeling the frequency response of the input impedance of a magnetic component based on a ferrite core that shows dimensional resonance at high frequency. The network was synthesized from a 5th-order Debye model obtained by empirical fitting to measurements [4]. Vesa et al. presented CNs for both the complex resistivity and the complex reluctivity of SMCs derived from analytical material models [5]. A major drawback of the existing CNs is that they cannot model the electromagnetic behavior of soft ferrites at high frequency, especially the resonance phenomena, while maintaining the dependencies of the microstructure.

The main goal of this work is to propose a practical approach for modeling SMMs at high frequency in the time-domain, while considering the heterogeneous microstructure. First, novel CNs are synthesized from the continued-fraction representations of analytical material models proposed earlier in [6]. The components of the proposed CNs maintain the dependencies of the physical and geometric properties of the material microstructure. The CNs are then coupled with 2-D

axisymmetric finite element (FE) models for simulating the electromagnetic behavior of SMM cores in both frequency- and time-domains. Numerical simulations are validated against measurements. Furthermore, ferrites and SMC cores are investigated under both dielectric and magnetic excitations.

II. METHODS

We present novel continued-fraction expressions for the effective behavior of the material and their corresponding Cauer networks. The proposed models are based on analytical material models developed earlier in [6]. The effective behavior of the material is then coupled with FE models for simulating the macroscopic electromagnetic behavior in both frequency- and time-domains.

A. Material models

The proposed continued-fraction expansion for the effective complex resistivity $\rho_{\text{eff}}(\omega)$ reads

$$\rho_{\text{eff}}(\omega) = \underbrace{\frac{1}{\frac{1}{\hat{R}_{-1}} + j\omega\hat{C}_{-1}}}_{\hat{\rho}_b(\omega)} + Z_0^{\text{RC}} + \underbrace{\frac{1}{\frac{1}{\hat{Z}_0^{\text{L}}} + \frac{1}{\hat{Z}_1^{\text{RC}} + \frac{1}{\frac{1}{\hat{Z}_2^{\text{L}} + \dots}}}}}_{\hat{\rho}_g(\omega)}, \quad (1)$$

$$\text{where } \hat{R}_{-1} = \frac{\tau}{(1+\tau)\eta\sigma_b}, \quad \hat{C}_{-1} = \frac{(1+\tau)\eta\epsilon_b}{\tau},$$

$$\hat{Z}_i^{\text{RC}} = \frac{1}{\frac{1}{\hat{R}_i} + j\omega\hat{C}_i}, \quad \hat{R}_i = \frac{1+2i}{(1+\tau)\eta\sigma_g}, \quad \hat{C}_i = \frac{(1+\tau)\eta\epsilon_g}{1+2i},$$

$$\hat{Z}_i^{\text{L}} = j\omega \underbrace{\frac{R_{\text{eff}}^2 \mu_g}{8(1+i)(1+\tau)\eta}}_{\hat{L}_i}, \quad (2)$$

subscripts g and b refer to the grain (particle) and its boundary (insulation), respectively, ω is the angular frequency, σ is the DC conductivity, and ϵ is the permittivity. Moreover, ζ is the relative boundary thickness, η is the volume fraction, μ is the

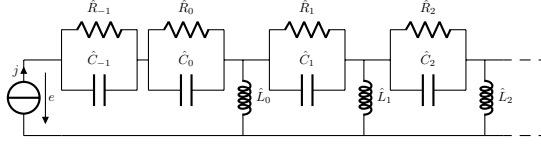


Fig. 1: Cauer network of ρ_{eff} .

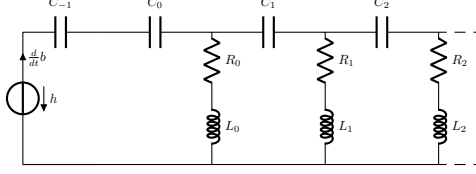


Fig. 2: Cauer network representation of $\frac{\nu_{\text{eff}}}{j\omega}$.

permeability, and R_{eff} is the effective grain radius. A novel CN is derived from the model in (1)-(2), and it is shown in Fig. 1.

It is not possible to derive a CN for the effective complex reluctivity $\nu_{\text{eff}}(\omega)$ with only passive circuit elements i.e., R , L , and C . Instead, a continued-fraction expansion for the quantity $\frac{\nu_{\text{eff}}(\omega)}{j\omega}$ is developed, and it reads

$$\frac{\nu_{\text{eff}}(\omega)}{j\omega} = \frac{1}{j\omega C_{-1}} + Z_0^C + \frac{1}{\frac{1}{Z_0^{\text{RL}}} + \frac{1}{Z_1^C + \frac{1}{Z_1^{\text{RL}} + \dots}}}, \quad (3)$$

where

$$C_{-1} = \frac{(1 + \tau)\eta\mu_b}{\tau}, \quad Z_i^C = \frac{1}{j\omega C_i}, \quad C_i = \frac{(1 + \tau)\eta\mu_g}{1 + 2i},$$

$$Z_i^{\text{RL}} = \underbrace{\frac{R_{\text{eff}}^2 \sigma_g}{8(1 + i)(1 + \tau)\eta}}_{R_i} + j\omega \underbrace{\frac{R_{\text{eff}}^2 \varepsilon_g}{8(1 + i)(1 + \tau)\eta}}_{L_i}, \quad (4)$$

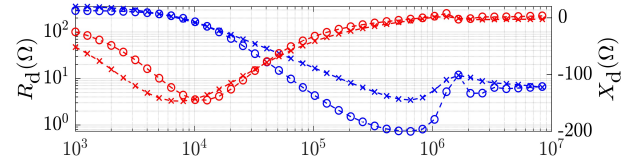
A novel CN is derived from the model in (3)-(4), and it is shown in Fig. 2. It can be seen from the models (1)-(4) that the components of the proposed CNs depend explicitly on the physical and geometric properties of the material microstructure, and there are no empirical parameters. Furthermore, considering the grain parameters σ_g and ε_g allows modeling the dimensional resonance phenomena at high frequency.

B. Macroscopic behavior in frequency- and time-domains

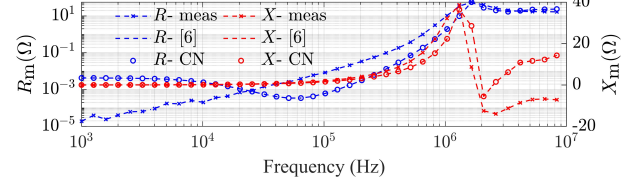
The macroscopic model presented here is the time-domain counterpart of the frequency-domain model presented in [6]. The following magnetodynamic model can be derived from the Ampère's and Faraday's laws

$$\nabla \times \mathbf{E}(\nabla \times \mathbf{H}) + \frac{\partial}{\partial t} \mathbf{B}(\mathbf{H}) = 0, \quad (5)$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic field strengths, respectively. It can be seen from (1) that the effective resistivity is a complex-valued parameter in the frequency-domain which is due to the fact that conduction and displacement currents are lumped into a total current in the frequency-domain. In the time-domain, the lumping results in a dynamically hysteretic $\mathbf{E} - \mathbf{J}$ model instead, which is tackled by the proposed CN



(a) Dielectric impedances.



(b) Magnetic impedances.

Fig. 3: Measured and simulated impedances of a toroidal core.

for the effective resistivity. The effective reluctivity is also a complex-valued parameter in the frequency-domain, therefore the $\frac{d}{dt} \mathbf{B} - \mathbf{H}$ model is used instead in the time-domain.

The finite-element method is used for solving (5) together with adequate boundary conditions. The boundary conditions depend on if the sample is excited by dielectric or magnetic excitation and whether the sample is a disk or a toroid. The magnetic field strength is given as a Dirichlet boundary condition, which ensures either a given magnetomotive force around a toroid or a given net current through a disk sample, depending on the type of excitation [6]. In addition, zero initial condition for the field strength is assumed.

III. RESULTS AND DISCUSSION

The proposed CNs and the values of the material parameters reported in [6] are used to obtain the frequency response of the effective behavior of a ferrite material. Then the macroscopic FE models are used for solving (4) in both dielectric and magnetic excitations for a toroidal core in the frequency-domain. The results are validated against the measurements and the simulations obtained by the models presented earlier in [6], and they are shown in Fig. 3. The simulations are in alignment, which is expected.

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