

Self-Adaptive Physics-Informed Neural Networks for Solving 2-D Magnetostatic Fields in Open Boundaries

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Open boundaries 2-D magnetostatic field problems pose significant challenges for traditional numerical methods due to their high computational complexity. This paper proposes MagOpenNet, a novel implementation of physical-informed self-adaptive neural networks (PINN) to solve these problems. MagOpenNet introduces the concept of the Infinite Element Method (IEM) and adjusts its architecture to address the complexities of predictions of the interior and exterior domains. The preliminary results state that MagOpenNet reduces 22.5% the computation time while maintaining 98% accuracy compared to traditional numerical methods.

Index Terms—Computational Electromagnetics, Open Boundaries, Magnetostatic, Neural Networks

I. INTRODUCTION

MAGNETOSTATIC field problems constitute a branch of computational electromagnetics that aims to investigate the magnetic fields within steady-state current systems. Traditional numerical methods such as the Finite Element Method (FEM) or Finite Difference Method (FDM), are usually employed to solve these problems. The handling of open or infinite boundaries poses significant challenges due to the extensive computational resources required. To address these, many approaches have developed the concept of a geometrically infinite finite element, evolving into the Infinite Element Method (IEM), showing its ability to solve open boundary problems [1], [2]. Recently, Physics-Informed Neural Networks (PINNs) have emerged as an alternative approach for solving magnetostatic problems [3], [4], offering advantages in handling complex geometries and maintaining accuracy through their ability to incorporate physical constraints. However, PINNs still struggle with open boundary conditions and computational efficiency. To address these limitations, we propose MagOpenNet, which stands for Magnetostatic Open Boundary Neural Network. It is a novel implementation of self-adaptive PINNs for solving 2-D Magnetostatic fields in open boundaries. MagOpenNet introduced the IEM and adjusted its architecture to address the complexities of predictions in the interior and exterior domains. Our contributions:

- Implements self-adaptive domain division for dynamic boundaries management
- Leveraged IEM to address high computational complexity in open boundary problems.
- Utilizes a hybrid loss function that combines physical constraints and geometric mapping.

II. METHOD

The magnetostatic problem with open boundary is divided into interior and exterior computational domains, with interface boundary Γ and boundary condition at Γ_∞ , as shown in Fig. 1. To identify the 2-D Magnetostatic boundary value problem,

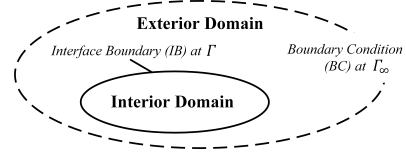


Fig. 1. The nomenclature for open boundary problem.

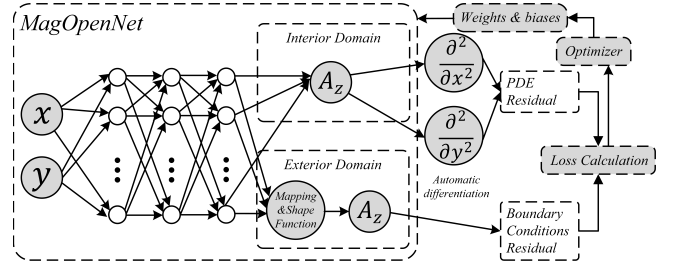


Fig. 2. Diagrammatic of the proposed MagOpenNet

some assumptions are made: the current density J_s and vector magnetic potential $\vec{A}$ are parallel with each other along the z -axis. The x and y components of the vector magnetic potential A_x and A_y are all zero, making A_z the principal component of the vector magnetic potential. The 2-D Magnetostatic boundary value problem can be expressed as:

$$\frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} = -\mu_0 J_s \quad (1)$$

The MagOpenNet draws the idea of infinite element technique, specifically designed to execute integral operations at the boundary denoted as Γ_∞ [6], [7], integrates multi dimensions configurations of the infinite element unit. The MagOpenNet also features the self-adaptive mechanism that dynamically selects the correct submodel for the interior or exterior domain and makes adjustments to the prediction to vary the sizes of the open boundaries. Fig. 2. provides the diagrammatic illustration of MagOpenNet. The 2-D Magnetostatic boundary value problem has been transferred to the corresponding residual formula. The MagOpenNet architecture aims to minimize this residual. The automatic differentiation function is utilized to

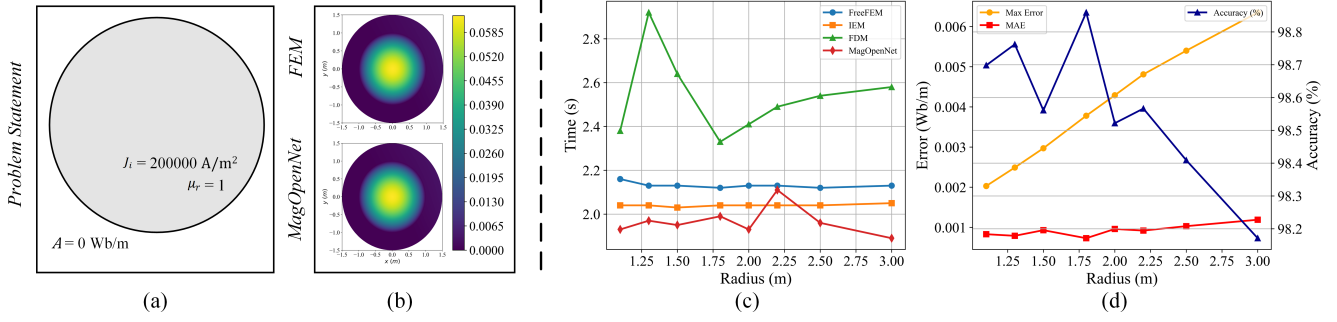


Fig. 3. Benchmark problems and comparison results. (a) Problem Statement. (b) Result by FreeFEM [5] and MagOpenNet when boundary computation halts at 1.5 m. (c) Comparison in computation time. (d) Comparison across error and accuracy.

approximate the solution for the interior domain and to then compute the residual of the PDE. The loss calculation in Fig. 2 proposed as a hybrid loss function:

$$L_{\text{total}} = w_1 L_{\text{PDE}} + w_2 L_{\text{BC}} + w_3 L_{\text{interface}} + w_4 L_{\text{decay}}, \quad (2)$$

where the weights are adaptively adjusts as:

$$w_i = \frac{\alpha_i}{\|L_i\|_2} \quad (3)$$

This loss function integrates the PDE residual loss (L_{PDE}), boundary condition loss (L_{BC}), interface continuity loss ($L_{\text{interface}}$), and decay loss (L_{decay}). The adaptive weighting mechanism dynamically balances these components, enhancing numerical stability and computational efficiency in solving open-boundary magnetostatic problems.

III. EXPERIMENT AND DISCUSSION

A. 2-D Magnetostatic in Circular Geometry

This study examines a 2-D Magnetostatic problem within a circular area of 1m radius, encapsulating an electrical cable constructed from a linear non-magnetic material such as copper, with a relative permeability (μ_r) of 1 for simplicity. A uniform current density has been determined at $2 \times 10^5 \text{ A/m}^2$. The problem is bounded by a Dirichlet condition, setting the magnetic potential A_z to zero at infinity, and adjusts the boundary computation halts ranges from 1.1m to 3m. The problem statement illustrated in Fig. 3.(a), MagOpenNet was developed based on Deepxde [8] and integrates 2-D infinite element configurations based on the formulation proposed by Tamitani in 2011 to validate the effectiveness of the proposed MagOpenNet through comparative analyses with state-of-the-art methods [1]. The benchmark methods comprise the FEM solved using FreeFem [5], the IEM based on Tamitani's boundary formulation, and a numerical implementation of the FDM. Performance evaluation metrics encompass computational efficiency through timing analysis, and FEM results serve as the reference standard for accuracy assessment, including maximum error, mean absolute error (MAE), and overall accuracy measurements.

B. Result statement

As depicted in Fig. 3.(b), MagOpenNet demonstrates its capability to solve designed 2-D Magnetostatic problems with

emphasized Dirichlet boundary conditions at the infinity domain. Fig. 3. (c) presents the computation time for various methods across traditional numerical methods and MagOpenNet. The results clearly show that MagOpenNet (red line) achieves the lowest computation time, leading FEM by an average of 7.74%, IEM by 3.62%, and FDM by 22.47%. Fig. 3.(d) illustrates the performance comparison between MagOpenNet and FEM in two indicators. The maximum error (orange line) increases slightly within a small range from 0.001 to 0.006. In contrast, the mean absolute error (MAE, red line) remains at a consistently low level, leading to an accuracy (blue line) that fluctuates within a range of 98.1% to 98.7%.

C. Discussion

The preliminary result indicates that the MagOpenNet not only outperforms traditional numerical methods in terms of computation time but maintains a comparable accuracy index. The changing computation boundary condition benchmark illustrates the self-adaptive ability of either the computation time or accuracy not to drop significantly. The detailed future work will explore the inclusion of 1-D and 3-D mapping functions, providing broader benchmark experiments and additional comparison indicators such as mean relative error.

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