

A Dynamic Energy-Based Hysteresis Model for Pulsed-Operated Fast-Ramping Magnets

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Due to the strongly nonlinear behavior of ferromagnetic cores, the numerical analysis of fast-ramping magnets is highly cumbersome and, therefore, in practice overly simplified by means of anhysteretic material descriptions and a posteriori loss formulae. This paper establishes a new dynamic ferromagnetic model combining a preconditioned energy-based hysteresis description and an eddy-current model in time domain. The model was successfully employed in the analysis of a normal-conducting bending magnet and succeeded to precisely calculate the losses and the time-lag between current excitation and air-gap field.

Index Terms—accelerator magnets, eddy currents, ferromagnetic materials, magnetic hysteresis, pulsed power systems

I. INTRODUCTION

FAST-ramping normal-conducting magnets delivering several kT/s receive increasing attention as they fill various key roles in particle accelerators, such as a synchrotron-based muon collider [1], [2]. Such magnets are excited by periodic pulsed-shaped currents with very short duty cycles [3], in order to minimize resistive losses while reliably enforcing the requested ramp-up of the magnetic field. The ramped excitation and the locally strong saturation of the ferromagnetic core prohibit the application of frequency-domain approaches. Moreover, calculating the hysteresis losses by post-processing an anhysteretic solution, e.g. by the Steinmetz-Bertotti formula [4] and according generalizations, yields inaccurate results as overall energy conservation and the time-lag between exciting current and generated magnetic field are neglected. In order to precisely resolve the ferromagnetic behavior of hysteresis and eddy-current phenomena, an adequate model will be formulated.

II. MAGNETIC CONSTITUTIVE LAW

The mapping $\mathbf{B} \mapsto (\mathbf{H}, \frac{\partial \mathbf{H}}{\partial \mathbf{B}})$ with magnetic flux density $\mathbf{B}$ and magnetic field strength $\mathbf{H}$ is essential for solving Ampère's law with a magnetic vector potential approach via a Newton iteration scheme. We begin with the macroscopic relationship

$$\mathbf{B} = (\mu_0 + \mu_0 \mathcal{M}) \mathbf{H}, \quad (1)$$

for which we introduce the magnetic polarization operator $\mu_0 \mathcal{M} : \mathbf{H} \mapsto \mu_0 \mathbf{M}$, which maps the applied magnetic field strength $\mathbf{H}$ to the reacting magnetic polarization $\mu_0 \mathbf{M}$. For air and vacuum, this mapping is $\mu_0 \mathcal{M} = 0$, while for slightly para- or diamagnetic materials linear models can be applied, e.g. for copper $\mu_0 \mathcal{M}_{\text{Cu}} = -9.6 \cdot 10^{-6} \mu_0$ at room-temperature.

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A. Ferromagnetic materials – Hysteresis model

The Bergqvist energy-based hysteresis model [5] refined by Henrotte [6] accounts for the hysteresis of ferromagnetic materials by expressing the magnetic polarization operator in terms of an anhysteretic polarization $\mu_0 \mathcal{M}_{\text{an}}$ and a number of history-dependent operators $\mathcal{H}_r^m$

$$\mu_0 \mathcal{M} = \sum_m w_m \mu_0 \mathcal{M}_{\text{an}} \mathcal{H}_r^m \quad \text{with} \quad \sum_m w_m = 1.$$

The operators $\mathcal{H}_r^m : \mathbf{H} \mapsto \mathbf{H}_r^m$ feature distinct pinning forces $\kappa_m \geq 0$ and store their previous states $\mathbf{H}_{r,\text{prev}}^m$. A play-model implementation provides a sufficiently good approximation of the operator $\mathcal{H}_r^m$, which must ensure the fundamental constraint of the energy-based model yielding non-negative hysteresis losses $p_{\text{hyst}}(\mathbf{H}, \mathbf{H}_r^m, \mu_0 \dot{\mathbf{M}}_m) \geq 0$ [6].

The required inverse mapping $\mathbf{B} \mapsto \mathbf{H}$ is more cumbersome to implement and most approaches rely on strongly relaxed iteration schemes [7]. Instead, we innovatively precondition (1) with the anhysteretic material description

$$(\mu_0 + \mu_0 \mathcal{M}_{\text{an}})^{-1} \mathbf{B} = (\mu_0 + \mu_0 \mathcal{M}_{\text{an}})^{-1} (\mu_0 + \mu_0 \mathcal{M}) \mathbf{H},$$

and introduce $\Psi(\mathbf{H})$ as the subtraction of right- and left-hand side. A handful fixpoint iteration steps without relaxation $\mathbf{H}^{\text{update}} = \mathbf{H} - \Psi(\mathbf{H})$ allow to reliably find $\mathbf{H}$.

B. Dynamic homogenized iron core model

In addition to hysteretic effects, the magnetic behavior of the laminated core is affected by eddy currents opposing the change of magnetic flux. The eddy-current effect can be considered by extending the material mapping to $(\mathbf{B}, \dot{\mathbf{B}}) \mapsto (\mathbf{H}, \frac{\partial \mathbf{H}}{\partial \mathbf{B}}, \frac{\partial \mathbf{H}}{\partial \dot{\mathbf{B}}})$ as

$$\mathbf{H} = (\mu_0 + \mu_0 \mathcal{M})^{-1} \mathbf{B} + \frac{\sigma d^2}{12} \dot{\mathbf{B}} = \mathbf{H}_{\text{avg}} + \mathbf{H}_{\text{eddy}},$$

with σ being the conductivity and d the lamination thickness [8]. The following analysis is based on this ferromagnetic model, which was combined with a two-dimensional (2D) finite element (FE) tool.

III. APPLICATION

A dipole magnet consisting of an angled H-type ferromagnetic core and eight solid conductors (Fig. 1) is analyzed. Its ferromagnetic core is manufactured of M235-35A10 steel laminates [9], [10] for which an approximated isotropic and anhysteretic material response is given by $\mu_0 \mathbf{M}_{\text{an}} = (\mu_0 M_1 L(|\mathbf{H}|/h_1) + \mu_0 M_2 L(|\mathbf{H}|/h_2)) \mathbf{e}_H$, with $\mu_0 M_1 = 1.39 \text{ T}$, $h_1 = 18.18 \text{ A/m}$, $\mu_0 M_2 = 0.56 \text{ T}$, $h_2 = 3.9 \text{ kA/m}$ and Langevin function L .

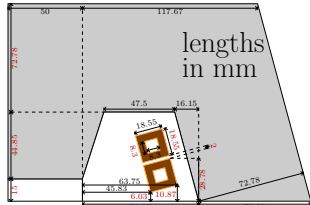


Fig. 1: One quarter of the analyzed H-type dipole magnet.

Firstly, a sinusoidal current of $12.5 \text{ kA} \sin(2\pi \cdot 500 \text{ Hz} \cdot t)$ per conductor was assumed. Figure 2 shows that the ferromagnetic losses resulting from the proposed model (black) are comparable to those of a standard Steinmetz-Bertotti approach with a posteriori loss estimates (blue), thus validating the proposed method. Additionally, a time-lag of $0.45 \mu\text{s}$ between current excitation and air gap field was found.

Secondly, an excitation of repeating triangular pulses (Fig. 3a) was considered. The resulting hysteresis loss of the proposed dynamic approach (Fig. 3b) is very similar to the one obtained with sinusoidal excitation. This is expected as the hysteresis loss mainly depends on the amplitude and is unaffected by the form and rate of the applied magnetic field. The eddy-current loss, however, differs substantially. The lower rate of the excitation slopes causes greatly reduced eddy-current losses of only 38.8 J/m for two cycles compared to 83.6 J/m for the sinusoidal current.

The dependency of the hysteresis curve on the excitation is visualized for a point with a predominant B_y component in Fig. 4. Due to larger eddy-current losses, the enclosed area of the sinusoidal excitation is larger than that of the pulsed excitation. We can also identify the different fall and rise rates of the triangular pulsed excitation since the losses accumulated on the ascending branch (right) are larger than those on the descending branch (left) whereas the sinusoidal area is well centered. We can also identify a plateau in the second quadrant where the B_y and H_y components remain once a pulse terminates.

IV. CONCLUSION

In this paper, an energy-based hysteresis model was combined with an eddy-current description to accurately model a fast-ramping magnet and its ferromagnetic core within a 2D FE framework. Consistency with traditional approaches has been shown for a sinusoidal excitation. Applicability for pulsed shaped excitations has been successfully demonstrated. The proposed model is indispensable if the time-lag between input current and air gap field or the losses of non-sinusoidal excitations have to be calculated.

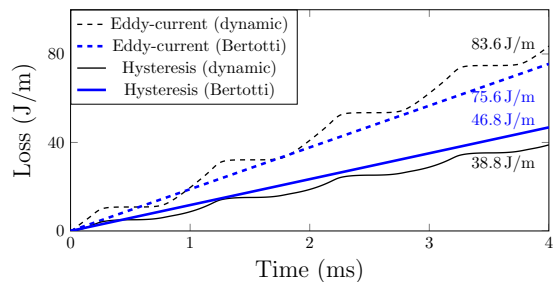
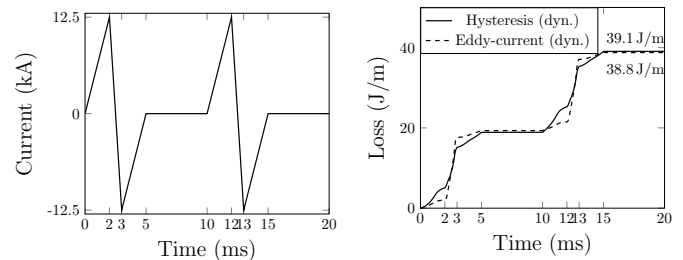


Fig. 2: Sinusoidal excitation: Accumulated hysteresis and eddy-current losses.



(a) Excitation current.

(b) Accumulated losses.

Fig. 3: Pulsed triangular excitation.

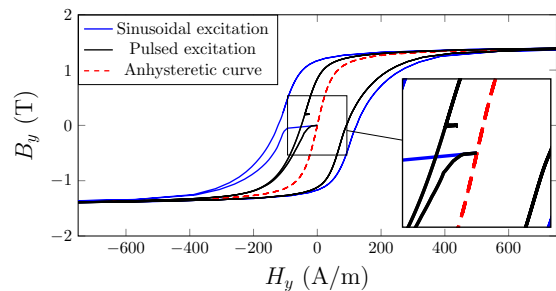


Fig. 4: Simulated ferromagnetic behavior at a selected position.

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