Modelling the Flux-Line Cutting in the Magnetization of a Weak-Pinning Type-II Superconductor

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The elliptic flux-line cutting model is extended beyond the critical state to describe the non-stationary magnetodynamics of a type-II superconductor subjected to a varying magnetic field. The Meissner currents, bulk pinning currents and flux cutting effects on the magnetic response of a rotating weak-pinning type-II superconductor, where the current density has perpendicular and parallel components to the magnetic field, is investigated. We perform numerical simulations to calculate the magnetic field in a type-II superconducting slab employing the method of lines and a second-order accurate spatial discretization based on a fixed set of nodes. For this system the numerical method is fast and accurate. Comparing our simulations with experimental data a quantitatively agreement was achieved.

Index Terms—Critical current density, Flux pinning, Magnetic hysteresis, Type II superconductors

I. INTRODUCTION

THIS DOCUMENT shows the effects of the Meissner surface currents, bulk pinning currents and flux cutting on the electrodynamics response of a type-II superconductor under a rotating magnetic field. Specifically, we numerically calculate profiles of the magnetic induction and the magnetic field as well as the magnetization. It is well established that in a non-ideal type-II superconductor subjected to small magnetic fields, close to the lower critical field $H_{c1}$, the magnetic dynamics is complex due to the interplay of the Meissner currents, the bulk pinning due to material defects, and the geometric barrier. Under these circumstances, the linear constitutive law $B = \mu_0 H$ is no longer valid and it is necessary to distinguish between the magnetic induction $B$ and magnetic field $H$. Here a novel nonlinear $B(H)$ relationship is proposed and tested.

II. THEORETICAL FRAMEWORK

Let us consider an infinite, isotropic type II superconducting slab $|x| \leq d/2$, $-\infty < y, z < \infty$ under an external magnetic field $H_0$. The magnetic field is written as $H = H\hat{e}_\alpha(\alpha)$, where $H$ denotes its magnitude and $\hat{e}_\alpha(\alpha) = \sin(\alpha)\hat{y} + \cos(\alpha)\hat{z}$ is a unit vector with $\alpha$ defined with respect to the $y$-axis. To express the current density $J$ and electric field $E$ in terms of their parallel and perpendicular components to $H$, we introduce the unit vector $\hat{e}_\perp(\alpha) = \cos(\alpha)\hat{y} - \sin(\alpha)\hat{z}$ perpendicular to $\hat{e}_\parallel(\alpha)$. Such vectorial notation is suitable for crossed or rotating external fields where the magnetic field and current density are not perpendicular to each other. Working at the H-formulation, the Maxwell’s equations for a slab geometry are

$$\frac{\partial E_\perp}{\partial x} + E_\parallel \frac{\partial \alpha}{\partial x} = -\frac{\partial B}{\partial H} \frac{\partial H}{\partial \alpha},$$

$$E_\perp \frac{\partial \alpha}{\partial x} - \frac{\partial E_\parallel}{\partial x} = -B(H) \frac{\partial \alpha}{\partial \alpha},$$

with the boundary conditions $H_\perp = H(x = 0, d)$.

We consider a constitutive relation between $E$ and $J$ such that their components, according to the elliptic critical-state model (1-2) are related by $E_\parallel = EJ_\parallel/J_{c\parallel}$ and $E_\perp = EJ_\perp/J_{c\perp}$. To take into account the superconducting nature of the sample, phenomenological models for the electric field $E$, the perpendicular critical current density $J_{c\perp}$ and parallel critical current density $J_{c\parallel}$ are required. We use the non linear relation $E = \rho J$ where the resistivity $\rho$ is modeled as

$$\rho(J, H) = \zeta_0 H \left(\frac{J/J_c}{1 + (J/J_c)^\sigma}\right)^\gamma,$$

the power $\sigma$ is the so-called creep exponent, $\zeta_0$ is a constant, and $J_c$ is the critical current density. The elliptical critical-state model incorporates the influence of the parallel component of the current density as follows:

$$J_c(H, \phi) = \left[\frac{\cos^2(\phi)}{J_{c\parallel}^2(H)} + \frac{\sin^2(\phi)}{J_{c\perp}^2(H)}\right]^{-1/2},$$

where $\phi(x)$ is the angle respect to the local magnetic field $H$. In this work we used $J_{c\perp} = J_{c\parallel}/(1 + H/H_c)^\gamma$ and $J_{c\parallel} = bJ_{c\parallel}$ with $b > 1$. To model the Meissner currents and the weak-pinning effects on the magnetic field dynamics we introduce a constitutive relation $B(H)$ adapted from [3]:

$$B(H) = \mu_0 \left\{ \begin{array}{ll} aH, & H \leq H_{c1} \\ aH + (H_{\gamma} - H_{c1})^{1/\gamma}, & H > H_{c1}, \end{array} \right.$$  \hspace{0.5cm} (5)

where $a \leq 1$ and $\gamma$ are parameters. The set (1)-(5) defines a non-linear partial differential equation that describes non-stationary states driven by the critical current density.

III. COMPARISON WITH EXPERIMENTAL DATA TO VALIDATE THE THEORETICAL PROPOSAL

We employed the method of lines to solve numerically the set (1)-(5), such methodology transforms the partial differential equation into an initial value ODE system problem by the spatial discretization of (2), using a second-order accurate spatial discretization based on a fixed set of nodes. We wrote the codes and ran our programs in the software MATLAB, in particular, to solve stiff differential equations the solver ode15s—a variable
has required. Since to their maximum value. good agreement between experiment and theory was achieved. PbIn seconds on a desktop workstation equipped with an Intel Xeon computing time to plot a hysteresis curve takes around 30 MATLAB function) that preserves the graph and therefore slow rotations in a static magnetic field H PbIn data for a Fig. 2. Rotational curves. The red curves M Fig. 1. The standard magnetization cycle. The red curve is the experimental M a good agreement between experiment and theory was achieved. Note that the numerical simulation starts at the first critical field Hc1 = 11.5 mT where there is vortex matter. For the flux-line cutting study, we considered an external magnetic field performing slow rotations (fixed xyz framework), such configuration is physically equivalent to a sample rotating in a fixed external magnetic field (mobile x'y'z' framework). At the fixed xyz framework the magnetization components are

\[
M_y = \frac{1}{\mu_0 d} \int_0^d \hat{e}_\perp(\alpha) \cdot B \, dx, \\
M_z = -H_a + \frac{1}{\mu_0 d} \int_0^d \hat{e}_\parallel(\alpha) \cdot B \, dx + M_{z,\text{Meiss}},
\]

where \(M_{z,\text{Meiss}}\) is the magnetization corresponding to the initial diamagnetic state.

Figure 2 shows the theoretical and experimental rotational curves. The experimental magnetization \(M_{y,E}\) and \(M_{z,E}\) were measured as the PbIn disk undergoes slow rotations in a static magnetic field \(H_a = 0.5 H_a \hat{z}\). The experiment started with the disk in a diamagnetic state. The blue curves \(M_{y,T}\) and \(M_{z,T}\) are the simulated magnetization using \(j_{\parallel} = 6 j_{c1}\). We appreciate a good agreement with the experimental results, therefore, the flux-line cutting is the main mechanism to describe the rotational curves. According to the experimental measurements, the behavior of both components is semi-periodic, the \(M_{z,E}\) component increases comparing with the initial diamagnetic state, while the \(M_{y,E}\) component behaves as a damped function. In addition to the flux consumption, we noticed a phase shift of the experimental \(M_{y,E}\) as a consequence that \(B\) is not aligned to the external field, this phenomenon produces a trapped magnetization at 180°, being even higher at the end of the period. We believe that such effect evidence that the superconducting material is slightly anisotropic.

**IV. Conclusion**

Even though the theory satisfactorily describes the magnetic response, there are some aspects that should be considered to improve the agreement with the experiment: 1) the demagnetizing field effect; 2) the finite geometry and; 3) how the initial diamagnetic state is reached. Therefore a deeper study of the flux-line cutting at this physical scenario is required.

**References**


