Sensitivity Matrix of an ECT System by Using FEA

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An electrical capacitance tomography (ECT) system is analyzed by a 2D FEM. Special focus is given to the sensitivity matrix evaluation in order to avoid the inherent numerical errors originated by the capacitance computation and by using the difference approximations for the Jacobian matrix. The FEM is applied to obtain the electric field. A new approach for the sensitivity matrix is developed by calculating capacitances and the Jacobian matrix using the own FEM formulation. Comparisons with published works were done.

Index Terms—Electrical capacitance tomography, electric field, finite element analysis, sensitivity.

I. INTRODUCTION

Electrical capacitance tomography (ECT) has been developed for imaging media containing dielectric materials [1].

Since the first ECT sensor in the late 1980s, ECT has been developed rapidly and used successfully till nowadays [2] in many applications, mostly for multiphase flow measurement [1], without excluding medical applications [3].

An ECT sensor consists of multiple measurement electrodes mounted equally around the cross-section of a vessel to be imaged [4] (see Fig. 1). For a sensor with \( N_n \) measurement electrodes, there are \( C(N_n,2) \) combinations of 2 from \( N_n \) independent electrode corresponding to equal number of independent capacitance measurements. In a complete measurement cycle, excitation electrode (1 to electrode \( N_n-1 \)) is chosen in sequence while the others, used as detection electrodes, are kept at ground potential. There are two major computational problems in ECT: the forward and the inverse. The forward problem consists to determine partial capacitances from the permittivity distribution by solving the partial differential fundamental equations of the electric field. The inverse problem is to determine the permittivity distribution from the capacitance measurements [5].

In this paper, for the forward problem, analysis of the sensing fields of ECT sensors is carried out using a 2D finite element method (FEM). Although the method can be easily extended to 3D field problems. When the ratio between the vessel diameter and the axial length of measurement electrodes is small enough 2D analysis can be accurate [4].

In this paper special attention is given to the sensitivity matrix determination. The sensitivity matrix is essential for the inverse problem solution [1, 4-6]. One way to solve the problem consists on the adjoint variable method (AVM) [7] where an adjoint equation is used to evaluate the adjoint variables which allows the Jacobian and sensitivity matrices to be obtained. In this paper, differently, a new approach for the sensitivity matrix evaluation is developed by using the inherent direct FEM formulation. Comparisons with published results show a good accuracy. The new method has the advantage in avoiding direct derivative calculations in two main steps: the gradient evaluation to obtain the electric charges and the numerical derivative calculations to obtain the Jacobian matrix.

Fig. 1. Cross section of the ECT system (a) where the sensing zone is partitioned into 15 regions. Generic region \( r \) is shown. Dimensions: inner vessel radius 10.5 cm; outer vessel radius 12 cm; screen radius 14 cm; electrode angle 32.14º. Medium relative permittivity: air 1; insulation 8; tomography chamber 1. Finite element discretization (b).

II. SENSITIVITY MATRIX

The application taken in this paper, used in [4], is represented in Fig. 1. The ECT sensor is composed by \( N_n=8 \) electrodes disposed symmetrically along the external surface of a cylindrical insulation. The sensing zone, occupying 1/8 of the chamber, is divided into \( N_r=15 \) homogeneous regions where the generic \( r \) region is represented. The coverage of the entire chamber is reached by rotating successively the sensing zone by \( \pi/4 \) which is equivalent just to change cyclically the active electrode. This subdivision is not essential, it allows the number of independent state variables (the permittivity) to be reduced. In the limit, each region could be coincident with the finite elements themselves. Each region is discretized into triangular finite elements represented in Fig. 1-(b) with \( N=3871 \) nodes and \( Ne=7152 \) finite elements. Due to the problem symmetry, it is sufficient to take the mode defined by imposing a voltage \( U \) to the electrode 1 (for instance) with respect to the grounded screen and take all other electrodes grounded allowing to obtain the capacitances \( C_{12}, C_{13}, C_{14} \) and \( C_{15} \) of the ECT sensor. These capacitances represents all the \( C(8,2)=28 \) capacitances of the system. The scalar electric potential satisfying the Laplace equation \((\nabla^2 \phi = 0)\) is evaluated using the FEM formulation. Homogeneous Dirichlet boundary conditions are imposed on the electrode boundaries, including the screen and excluding electrode 1 where the voltage \( U \) is applied.

FEM formulation gives the following matrix relation

\[
[M]([\phi]) = (0), \quad [M] = [\epsilon][G]
\]

where the permittivity matrix \([\epsilon] \) is a block diagonal matrix of order \( N \times (N \times N_r) \) where the diagonal element of order \( i=1, \ldots, \)
N for the region \( r=1,\ldots,N_r \) is the region permittivity if the node \( i \) belongs to an element of \( r \) and null in all other cases. The matrix \([G]\) is a matrix of order \((N \times N_r) \times N\) where the element of block \( k \) for the region \( r=1,\ldots,N_r \) and column \( j, k, j=1, \ldots, N \), is given by

\[
G_{ij}^k = \sum_{e \in \Omega} q_{ij}^e, \quad g_{ij}^e = \int_{\gamma_e} (\nabla \varphi_i^e \cdot \nabla \varphi_j^e) dS \quad k, j, e
\]

and 0 for all other cases, where the index \( e \) denotes the order of a finite element, the functions \( \varphi \) are the basis functions (in this case being polynomial interpolation functions of order 1). The matrix relation (1) is reduced to take into account the Dirichlet nodes allowing the following equation matrix to be obtained

\[
[M]_i^R(\phi)_R^e = (H)_i^e, \quad [M]_i^R = [e]_i^R [G]_R^e
\]

where the index \( R \) indicates reduced matrices. Now \([e]_R^e\) is of order \( N^e \times (N^e \times N_r) \), \([G]_R^e\) is of order \((N^e \times N_r) \times N^e\) and the column matrices \((\phi)_R^e\) and \((H)\) are of order \( N^e \times 1 \) where \( N^e = 3267 \) is the number of unknowns (removing the Dirichlet node equations) and where \((H)\) takes into account the terms associated with the Dirichlet node potentials.

Noting that, for each element \( e \), the FEM formulation implies that

\[
\sum_{k} \int_{\gamma_e} (\nabla \varphi_i^e \cdot \nabla \varphi_j^e) \varphi_j^e dS = \int_{\gamma_e} \left( \frac{\partial \varphi_j^e}{\partial n} \right) dS = q_{ij}^e
\]

where \( \gamma_e \) is the boundary of the element \( e \) with cross section \( S_e \). Then, electric charge (per unit length – p.u.l.) on the electrode \( n \) with boundary \( s_n \) may be given by

\[
q_n = \int_{\gamma_n} \left( \frac{\partial \varphi_j^e}{\partial n} \right) dS = (M^e_n)^T (\phi), \quad (M^e_n)^T = \sum_{i=1}^{N_e} (M_i)^T
\]

where\( (M_i)^T \) is the line matrix of order \( i \) of the square matrix \([M]\) of (1).

The extension to 3D field problems implies that the 2nd member of (4) is a surface integral over the boundary of the volume element \( e \) and (5) corresponds to the displacement vector flux across the conductor \( n \) surface.

The partial capacitance \( C_{ij} \) p.u.l. is defined as \( q_{ij}/U \) and then the set of partial capacitances \( C_{ij}, i=2,\ldots,N \) is given by

\[
(C) = -(1/U)[M^T](\phi)
\]

\([M^T]\) being of order \((N-1) \times N\), built by concatenation of the line matrices \([M_i^T]\), \( i=2,\ldots,N \), the variation of \( (C) \) caused by a permittivity variation in the sensing zone has then the form

\[
(\Delta C) = -(1/U)[M^T]\frac{\partial [\phi]}{\partial \varepsilon_k} \quad (\Delta [\phi] = [J]\Delta C)
\]

where \([J]\) is the Jacobian matrix of order \( N \times N_r \), for linear variation assumption. The result is therefore valid for small variations around the given permittivity values. The matrix \([M^T]\) does not vary with the permittivity because their elements are associated with the region adjacent to the electrodes which is the insulation wall and there is no variation for the Dirichlet nodes. Taking into account (3), the column \( j \) of \([J]\) is given by

\[
(J_j) = -[M]^T_\varepsilon \frac{\partial \varepsilon_k}{\partial \varepsilon_j} [G]_R^e(\phi)_R^e
\]

The sensitivity matrix \([S]\) is finally given by

\[
[S] = -(1/U)[M]^T[J], \quad (\Delta C) = [S]\Delta \varepsilon
\]

III. Results

Results were obtained for the ECT system analyzed in [4] represented and described in Fig. 1 (a). The forward problem was solved when electrode 1 is excited taking the relative permittivity \( \varepsilon_r = 1 \) inside the tomography chamber. The partial capacitances normalized by \( \varepsilon_0 \) are found to be: \( C_{12}=C_{18}=3.713 \), \( C_{13}=C_{17}=0.116 \), \( C_{14}=C_{16}=0.059 \), \( C_{15}=0.049 \). Deviations less than 1% were obtained comparing with a commercial FEA software. The sensitivity matrix was evaluated where the results for \( C_{12} \) and \( C_{15} \) are represented in Fig. 2 showing their variation with each region of the sensing zone rotated successively to fill the entire chamber. Results agree with the ones of the bibliography [4-5,7] showing the same variation and behavior.

Fig 3 shows the variation of \( S_{12} \) with its permittivity at the cross region of Fig.1 for two electrode lengths with the values of 32.16\(^o\) and 40\(^o\). Results show the typical non linearity of capacitance and sensitivity and show also that the sensitivity tends to increase with the electrode length.

ACKNOWLEDGEMENTS

This work was supported in part by Fundação para a Ciência e Tecnologia (FCT)/MCTES through Portuguese national funds and when applicable co-funded by EU funds under the project UIDB/50008/2020.

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