A New Algorithm for Tracing Flux Lines of HVDC Ionized Field Based on Bezier Curves

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A new method for tracing the electric flux lines in the flux tracing method (FTM) is developed by using a parametric curve. The flux lines can be traced without accumulating errors caused by numerical differentiation for the electric potential. The control points of Bezier curve are modified to minimize an error function defined by the electric field vector. Once the electric flux lines are traced, a series of flux tubes is created by bundling adjacent flux lines, and the nodal electric field and space charge distribution are iteratively updated. New flux lines are traced by the modified electric field, and the process is repeated until the effect of the space charge is fully considered.

Index Terms—Bezier curve, corona, flux tracing method, HVDC, ion flow field

I. INTRODUCTION

The space charge emitted from a coronating conductor surface flows along the flux line of the electric field distribution. Since the flux lines specifies the transport direction of the space charges, the problem is simplified by a series of 1D equations and can be solved along each flux line. The flux tracing method (FTM) [1] introduced a field modification factor to express the electric field vector in the presence of space charge. Based on this so-called Deutsch’s assumption, the electric field and space charge distributions can be described with respect to the electric potential variation along the flux line. In recent years, iterative FTM are proposed in [2]-[3] to reduce the error on the electric field due to Deutsch’s assumption.

In order to take the effect of space charge distribution into consideration without Deutsch’s assumption, the flux lines in conventional FTM are iteratively traced to be orthogonal to the equipotential. The calculations for spatial potentials involve a first order derivative so that each iteration can accumulate a significant amount of error due to numerical differentiation, resulting in divergence of the solution.

In this paper, a new method for FTM using a parametric curve is presented. The flux lines are traced based on an error function defined by electric field vectors.

II. FORMULATION

A. Assumptions and basic equations

In general, the corona discharge and ionization process can be mathematically described by making several simplifying assumptions [4]. In the specific case of unipolar corona, the nonlinear relation between the electric field and ionic space charges can be obtained by solving the following equations.

\[ \nabla \cdot \mathbf{E} = \rho / \varepsilon_0 \]  
\[ \mathbf{J} = \mu \rho \mathbf{E} \]  
\[ \nabla \cdot \mathbf{J} = 0 \]

where \( \mathbf{E} \) and \( \mathbf{J} \) are the electric field and the current density vectors at any point in space, \( \varepsilon_0 \) the permittivity of free space, \( \mu \) the ionic mobility, and \( \rho \) the space charge density.

The space charge distribution is influenced by the electric field generated by the induced voltage on the conductor, and the modified space charge distribution affects the existing electric field in space. In the iterative scheme, both \( \mathbf{E} \) and \( \rho \) are updated to satisfy (1) to (3). In this paper, the effect of the recombination, ion diffusion, and wind is neglected for simplification.

B. Flux lines based on parametric curve

Since the electric field distribution is continuous and smooth in the inter-electrode region, the flux lines can be represented by parametric curves that are assumed to originate from the conductor surface and terminate on the ground plane or the other conducting surfaces with the opposite polarity.

Bezier curve is a space curve, which is contained within the convex hull of the Bezier polygon defined by several control points. Due to the fact that the tangent vectors at each end are directed along the first and last span of the polygon, conditions for the electric field on the conductor surface and ground plane can be easily satisfied.

A parametric Bezier curve of degree \( M \) is defined as

\[ \mathbf{B}(\tau) = \sum_{i=0}^{M} C_i^M (1-\tau)^{M-i} \tau^i, \quad 0 \leq \tau \leq 1 \]

where \( \tau \) is a parameter, \( i \) the summation index, \( C_i^M \) the binomial coefficient, and \( \mathbf{P}_i \) the \( i^{th} \) control point. In a conductor-plane configuration, the first point \( \mathbf{P}_0 \) and the last point \( \mathbf{P}_M \) are subject to be located on the conductor surface and on the ground, respectively. The derivative of (4) with respect to \( \tau \) is

\[ \mathbf{B}'(\tau) = \sum_{i=0}^{M-1} M (\mathbf{P}_{i+1} - \mathbf{P}_i) b_{M-i,i} \]

where \( b_{M,i} \) is Bernstein basis polynomials of degree \( M \) and is equal to \( M C_i^M (1-\tau)^{M-i} \tau^i \). Since \( \mathbf{B}' \) is supposed to represent the electric flux line, \( \mathbf{B}' \) should be tangential to the electric field vectors at any point along the curve. In addition, \( \mathbf{B}'(0) \) and \( \mathbf{B}'(1) \) must be normal to the conductor surface and ground plane, respectively.

Based on the derivatives and electric field vectors along the
curve, an error function can be defined as follows.

\[ \varepsilon = \left( 1/2 \right) \left( 1 - \frac{\mathbf{E} \cdot \mathbf{B}'}{\mathbf{E} \cdot \mathbf{B}'} \right)^2 \]  

(6)

The error function above represents the difference in direction between the tangential components of the curve and electric field vectors at a specific point. Therefore, the error defined in (6) should be minimized close to zero.

The space-charge-free electric field at any point in space can be evaluated from the surface charge density generated by the conductor voltage. The unknown surface charge density can be calculated by the method of moment (MoM), which can significantly reduce the computational load by concentrating the region of interest on the conductor surface where the charge can exist.

The process for tracing the flux line may be described by the following steps.

Step 1) An initial flux line is assumed by (4) with \( \mathbf{P}_0 \) fixed at a specified point on the conductor surface and \( \mathbf{P}_M \) located on the ground plane. \( \mathbf{P}_1 \) and \( \mathbf{P}_{M-1} \) are set to satisfy the boundary conditions on the conductor surface and ground plane.

Step 2) The derivative of the curve is calculated by (5).

Step 3) The error is computed along the curve by (6).

Step 4) The control points are modified to minimize the error.

Step 5) The flux line is traced and represented by the Bezier curve with new control points modified in Step 4).

The preceding five steps are repeated for different points around the conductor surface to obtain the entire flux lines in the region of interest. For a unipolar DC line conductor 0.1 m in radius suspended 2 m above a ground plane, the traced flux lines are shown in Fig. 1.

Due to the fact that the flux lines are traced based not on the electric potential but on the electric field itself, errors caused by numerical differentiation can be minimized, especially near the conductor surface where the potentials change very rapidly.

To minimize the error function in Step 4), various optimization techniques such as the least square method (LSM), the pattern search method (PSM), and the steepest descent method (SDM) can be utilized. In this paper, the SDM-based algorithm is used to determine the control points, and the formulation for the error minimization will be presented in detail in the extended version.

C. Current flow model for ion flow field

Once the electric flux lines are traced, a series of flux tubes can be created by bundling adjacent flux lines. Since the space charges emitted from the conductor surface flow along the flux lines, the ion flow in the flux tube should be conservative.

To construct a current-flow model, each flux tube is subdivided into triangle elements, and the initial values for the electric field and space charge density are assigned to each node. Using the integral form of (3) and the Gauss’ divergence theorem, an edge current for a triangle element can be defined in terms of nodal current densities as

\[ \hat{\mathbf{J}}_i \cdot \mathbf{ds} = \sum_{n=i,j,k} \mathbf{I}'_n = 0 \]  

(7)

\[ \mathbf{I}'_n = \frac{1}{2} \left( \mathbf{n}_i \cdot (\mathbf{J}_i \hat{\mathbf{e}}_i + \mathbf{J}_j \hat{\mathbf{e}}_j) \right) \]  

(8)

where \( \mathbf{J} \) is the magnitude of the nodal current density, \( \hat{\mathbf{e}} \) the unit electric field vector, \( \mathbf{I} \) the edge current flowing into or out of the element along the edge normal vector \( \mathbf{n}_i \), \( l \) the length of the \( l \)th side, and the subscript \( i,j,k \) the node numbers.

An iterative calculation is conducted with the initial space charge densities until the Townsend’s assumption [5] is reached. In each iteration for updating nodal charge density, the electric field vector is assumed to be constant at each node. Once the space charge distribution is calculated, the electric field in (6) is updated to trace a new set of flux lines, and it will be repeated iteratively until the deviations of flux lines are within acceptable limits. In Fig. 2, the calculated charge distribution for a concentric cylinder with the inner radius of 0.1 m and the outer radius of 1 m is shown with the flux lines.

The numerical analysis will be presented in detail in the extended version. In addition, the FTM analysis for the conductor-plane configuration will be also presented.

REFERENCES