

# Computation of Magnetic Contact Forces

Jangho Seo<sup>1</sup> and Hong Soon Choi<sup>2</sup>

<sup>1</sup>School of Automotive Engineering, Kyungpook National University, Sangju 742-711, Korea

<sup>2</sup>Department of Electrical Engineering, Kyungpook National University, Daegu 702-701, Korea  
tochs@knu.ac.kr

**Abstract**—This paper presents a general scheme for computation of contact forces when magnetic materials are touching each other. Maxwell stress tensor method is adopted by using the virtual air-gap scheme. By using the scheme, general contact situations such as iron versus iron, iron versus magnet, magnet versus magnet, irregular shaped objects, and high field saturation can be resolved in a consistent way. The proposed scheme is easily applicable by just implementing in post-processing step. Numerical results are shown for the validity.

**Index Terms**—Magnetic forces, Maxwell stress tensor, contact forces, virtual air-gap, finite element analysis.

## I. INTRODUCTION

Computation of magnetic forces between two magnetic material bodies in contact is an essential need in electric machine designs. For example, in large-scale electric motors, the estimation of contact force between an assembly of permanent magnet with iron bed and rotor yoke is a decisive factor for determining a required clamping force of the bolt used. It is also important when predicting absorptive force of electromagnet, bonding strategy of the magnet and yoke, etc. As far as we know, there is no general finite-element-based software resolving these contact problems. So users might solve the problems by intentionally adding a thin air-gap between the contact objects. The added gap deteriorates the quality of solution because of the unnecessary distance and the distorted shape of elements in the gap.

In recent years, some studies to overcome the problem have been delivered as follows. By using virtual work principle, a method of shell elements, which are introduced on the boundary of objects, was proposed by Ren and Cendes [1]. After taking the derivative of the co-energy, the thickness of shell element is reduced to zero. This approach has the accuracy problem when the touched objects are different in their permeability because of no explicit field expression inside the shell. More recently, for computing the contact forces of a permanent and an iron plate, an analytic solution was shown by using the image method [2]. And dual dipole model of magnetic material was presented for calculating the force of subvolumes of material bodies with limited contact situation [3].

In this paper, for providing a general computation scheme of magnetic contact forces, Maxwell stress tensor method is adopted while slightly modifying the property of the integration surface. It is applied by using the fields inside the virtual air-gap [4] derived from the generalized magnetic source methods [5]-[7] in the contact surfaces. In the references [4]-[7], the virtual air-gap application of Maxwell stress tensor was only dealt with in a way of conceptual approach. It is noted that the derived fields of the virtual air-gap are on the boundary, not inside of it [5]. It is better to adopt the fields inside than those on the boundaries because

the integration path of Maxwell stress should be taken inside the gap for obtaining improved accuracy. The solutions in the previous works are neither widely accepted nor even perceived [2] because they provide limited scope or are rather complex to be managed to get contact forces. To resolve this situation, this paper presents an easily applicable, general and robust scheme using one of the most commonly used force calculation methods, i.e., Maxwell stress tensor.

The proposed scheme could be implemented by adding a simple procedure in post-processing step when computing the force. That is, there is no need to modify any material modeling or finite element formulation. If the scheme is implemented, general contact situations such as iron versus iron (might having different permeability), iron versus magnet, magnet versus magnet, irregular shaped objects, and high field saturation could be resolved in a consistent way. In this paper, a model of electromagnet is adopted for showing the validity of the contact force calculations. Numerical results of the contact force according to the applied current when in contact and the decreasing force when the attracted magnetic object goes apart from the electromagnet are presented. In the full paper, numerical results and experimental measurements with various combinations of magnetic materials having different permeability and sizes will be shown.

## II. MAXWELL STRESS ON CONTACT SURFACE

As seen in Fig. 1, for calculating total force of the magnetic material 2, the field expression of the virtual air-gap is needed. Based on equivalent magnetic sources [4][5], the field intensity inside the virtual air-gap is derived as

$$\mathbf{H}_{\text{vgap inside}} = H_n \mathbf{n} + H_t \mathbf{t} = \frac{\mathbf{H}_{\text{vgap from 1}} + \mathbf{H}_{\text{vgap from 2}}}{2}. \quad (1)$$

In the above expression,  $\mathbf{H}_{\text{vgap from 1}}$  and  $\mathbf{H}_{\text{vgap from 2}}$  are respectively

$$\mathbf{H}_{\text{vgap from 1}} = \frac{B_{1n}}{\mu_0} \mathbf{n} + H_{1t} \mathbf{t}, \quad (2)$$

and

$$\mathbf{H}_{\text{vgap from 2}} = \frac{B_{2n}}{\mu_0} \mathbf{n} + H_{2t} \mathbf{t}. \quad (3)$$

where the index number means material's number;  $\mathbf{n}$  and  $\mathbf{t}$  are a normal and a tangential unit vector respectively;  $\mu_0$  is air permeability;  $B_{1n}$ ,  $H_{1t}$  are, respectively, normal component of flux density and tangential component of field intensity in the material 1;  $B_{2n}$ ,  $H_{2t}$  are fields in the material 2. By using (2) and (3), the field intensities inside virtual air-gap,  $H_n$  and  $H_t$ , in (1) are given as follows,

$$H_n = \frac{B_{1n} + B_{2n}}{2\mu_0}, \quad H_t = \frac{H_{1t} + H_{2t}}{2}. \quad (4)$$

This virtual air-gap field extracted from each material,  $\mathbf{H}_{\text{vgap from 1}}$  or  $\mathbf{H}_{\text{vgap from 2}}$ , is identical with the field that is estimated from the inside field of the material as it satisfies the boundary conditions, which are equalities of normal component of flux density and tangential component of field intensity, as if there is an air space between the material 1 and 2. Maxwell stress  $\mathbf{f}_s$  is written as

$$\mathbf{f}_s = \frac{1}{2}\mu_0(H_n^2 - H_t^2)\mathbf{n} + \mu_0 H_n H_t \mathbf{t}. \quad (5)$$

By surface-integrating  $\mathbf{f}_s$  in the closed surface which includes virtual air-gap surface, the total force of the object could be obtained.

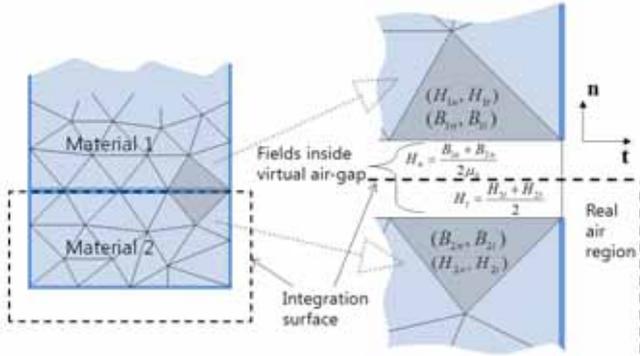


Fig. 1. Schematic of virtual air-gap application. The virtual air-gap is taken as the integration surface for Maxwell stress tensor method. The virtual air-gap fields are, as a rule, varying along with integration path and dependent on the adjacent elements. The gap is just virtually imagined, not really created in the domain.

### III. NUMERICAL TEST

A numerical electromagnet model and a resultant flux pattern are shown in Fig. 2.

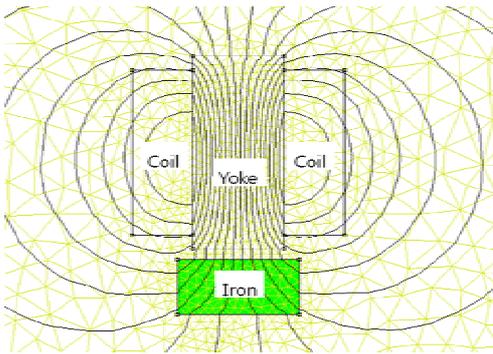


Fig. 2. Electromagnet 2D model. Unit: mm. Yoke: height 70 x width 30, Iron: height 20 x width 40. Number of coil turns: 5000.

Total forces of the iron according to air-gap length are shown in Fig. 3. When the air-gap length is zero, the virtual air-gap scheme was used in the contact surface. A meaning of this result is that the fields in air-gap converge to those of

virtual air-gap when the iron approaches the yoke. In Fig. 4, the contact forces of the iron according to applied current magnitude are shown. These successful numerical results show the validity of the proposed method. In the full paper, a comparison between numerical results and experimental measurements will be shown with various combinations of magnetic materials.

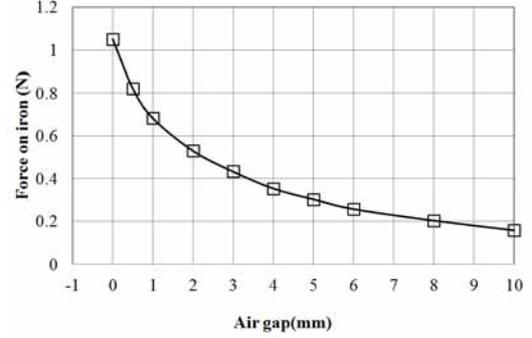


Fig. 3. Total forces of iron according to air-gap length. When air-gap is zero, virtual air-gap was used in the contact surface. Current 0.3A applied.

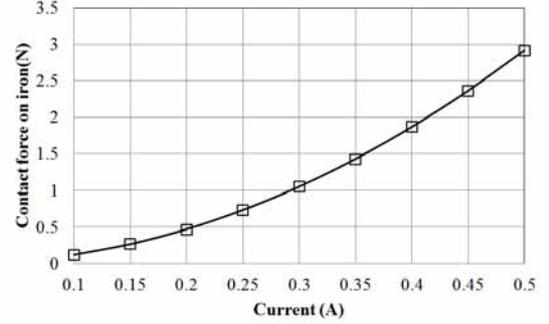


Fig. 4. Contact forces of iron according to applied current. There is no air-gap when in contact.

### REFERENCES

- [1] Z. Ren and Z. Cendes, "Shell elements for the computation of magnetic forces," *IEEE Trans. Magn.*, vol.37, no.5, pp.3171-3174, 2001.
- [2] M. Beleggia, D. Vokoun, and M. D. Graef, "Forces between a permanent magnet and a soft magnetic plate," *IEEE Magnetics Letters*, vol.3 (2012), 0500204
- [3] T. Kovanen, T. Tarhasaari, and L. Kettunen, "Localization of electromagnetic force based on material models," *IEEE Trans. Magn.*, vol.48, no.1, pp.13-17, 2012.
- [4] H. S. Choi, S. H. Lee, Y. S. Kim, K. T. Kim, and I. H. Park, "Implementation of virtual work principle in virtual air gap," *IEEE Trans. Magn.*, vol.44, no.6, pp.1286-1289, 2008.
- [5] H. S. Choi, I. H. Park, and S. H. Lee, "Concept of virtual air gap and its application for force calculation," *IEEE Trans. Magn.*, vol.42, no.4, pp. 663-666, 2006.
- [6] H. S. Choi, S. H. Lee, and I. H. Park, "General formulation of equivalent magnetic charge method for force density distribution on interface of different materials," *IEEE Trans. Magn.*, vol.41, no.5, pp. 1420-1423, 2005.
- [7] H. S. Choi, S. H. Lee, and I. H. Park, "Generalized equivalent magnetizing current method for total force calculation of magnetized bodies in contact," *IEEE Trans. Magn.*, vol.42, no.4, pp.531-534, 2006.