A New Numerical Scheme for the Simulation of Corona Fields

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***Abstract*—A new numerical scheme is proposed for the computation of the corona space charge density and electric field distributions in wire-duct electrode configurations. We use a technique based on a combination of finite element and donor cell methods (FEM-DCM). The solution procedure is obtained iteratively by using the Newton-Raphson algorithm in order to converge to a self-consistent solution. An electric vector potential formulation is used for the computation of the electrostatic field whereas the current continuity equation is applied for the derivation of the space charge density. The same mesh is used throughout the whole iterative procedure, the Kaptzov condition is easy to handle and there is no need for an outer loop in order to impose the correct charge condition on the surface of the corona wire electrode.**

***Index Terms*—electric corona discharges, finite element methods, numerical simulation.**

I. Introduction

The electric corona discharge is of common use in numerous engineering applications such as electrostatic precipitation [1], decomposition of toxic gases, ozone generation and others. However it can also happen as an unwanted phenomenon such as in high voltage transmission overhead lines where it is responsible of power losses that must be avoided. In any case, it is important to have an accurate modeling in order to either optimize the devices or minimize the losses in transmission grids.

Simulation of the electric corona discharge is not obvious since the related set of equations contains a nonlinear coupling between the electrostatic field and space charge quantities. Both of them depend on each other so that an iterative scheme is needed to find a self-consistent solution. The classical formulation for the electric field problem is the Poisson equation which describes the electric scalar potential under suitable boundary conditions and a given space charge distribution. A particular additional boundary condition related to the corona effect must also be considered. That is the constancy of the value of the normal electric field on the corona electrode for voltages beyond the corona inception level (Kaptzov condition). The field is then equal to the onset value E_0 obtained from Peek's formula. The finite element method (FEM) is the most used technique for the numerical solution of the electrostatic field problem [2], [3]. As the electric space charges move, the charge problem is governed by the current continuity condition that allows the computation of the charge distribution under the electric field computed in the field problem. Several techniques have been developed for the space charge density problem such as the method of characteristics (MOC) [3] and the donor-cell method (DCM) [4]. This last technique is a upstream finite volume scheme and it will be used here in combination with the FEM technique applied to an electric vector potential formulation instead of the traditional scalar one.

The corona electrode configuration is often of wire type and therefore it will be considered in this paper. In this situation the Kaptzov condition is very easy to handle since the electric flux is now applied as the essential boundary condition instead of the scalar electric potential of the electrodes. The use of DCM is also more direct because it is directly applied to the FEM mesh and no Voronoi pattern is required as in [4]. The Newton-Raphson algorithm is employed for the convergence to a self-consistent solution. Two-dimensional problems are considered as it is generally the case in this context but the extension to 3-D problems is straightforward.

II. Problem Formulation

The governing equations of the corona problem are

$$\nabla . \boldsymbol{D} = \rho \tag{1}$$

$$\nabla \times \boldsymbol{E} = 0 \tag{2}$$

$$\nabla . \boldsymbol{J} = 0 \tag{3}$$

with the constitutive relations

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} \tag{4}$$

$$\boldsymbol{J} = k\rho \boldsymbol{E} \tag{5}$$

where $\boldsymbol{D}$ is the electrostatic displacement, ρ is the space charge density, $\boldsymbol{E}$ is the electric field, $\boldsymbol{J}$ is the electric current density, ε_0 is the gas permittivity and k is the mobility of charge carriers.

We introduce the electric vector potential $\boldsymbol{P}$ defined in accordance to (1):

$$\boldsymbol{D} = \boldsymbol{D}_s + \nabla \times \boldsymbol{P} \tag{6}$$

where $\boldsymbol{D}_s$ is a source electric flux density satisfying (1) but not (2) *a priori* so that a gauge condition must be applied. Combining (1), (2) and (4), the following (translational) 2-D div-conform formulation is obtained:

$$\nabla^2 P = \left(\frac{\partial D_{sy}}{\partial x} - \frac{\partial D_{sx}}{\partial y} \right) \quad \text{on} \quad \Omega \tag{7}$$

with the boundary conditions

$$P = P_0 \ , \boldsymbol{n}.\boldsymbol{D}_s = 0 \quad \text{on } \Gamma_d \quad \text{and} \quad \boldsymbol{n} \times \boldsymbol{D} = 0 \quad \text{on } \Gamma_e \tag{8}$$

Domain Ω is the inter-electrode space, Γ_e relates to the conductor boundaries and Γ_d is the remaining part. The Dirichlet boundary condition on Γ_d allows the specification of the electric displacement flux on the corona electrode. Note that for 2-D problems, $\boldsymbol{P}$ reduces to its z-component P and naturally satisfies the Coulomb gauge ($\nabla.\boldsymbol{P} = 0$).

The use of the FEM requires consistent discretization spaces for the variables. Quantities P and $\boldsymbol{D}_s$ (and also $\boldsymbol{D}$) will be decomposed in nodal and "facet" element spaces respectively, related to 2-D problems. The space charge density ρ is implicitly discretized in the "volume" element space. Linear triangles are assumed for the mesh. A classical tree technique is used in order to define the source field where the flux of $\boldsymbol{D}_s$ is fixed at 0 on each edge of the tree (i.e. the gauge $\boldsymbol{D}_s.\boldsymbol{w} = 0$). The fluxes on the co-tree edges result from the Gauss's law applied to $\boldsymbol{D}_s$ for the associated loops. An electric displacement field is then derived for a given space charge density in the inter-electrode interval from (7) and (6).

The space charge density problem is solved by a upstream finite volume technique, the DC-method, applied by combining (3), (4) and (5) as in [4] but in a specific way since now we consider the same finite element mesh instead of the classical dual Delaunay-Voronoi polygon tessellation. The current which crosses a "facet" (i.e. an edge) of any triangle t is the flux of $\boldsymbol{J}$ directly obtained from the fluxes of $\boldsymbol{D}$ by using (4) and (5). The charge density ρ of the adjacent triangle is used for edges for which current enters the triangle t and charge density inside the triangle t is used for edges for which current leaves the triangle. This upstream difference method requires fixing the space charge density on the triangles along the corona electrode. Writing the current conservation for each triangle yields a system of linear equations which can be solved to determine the charge density distribution for a given electric displacement field.

Finally, self-consistent distributions for both $\boldsymbol{D}$ and ρ are obtained by considering volume charge density and node potential values simultaneously as unknowns, and using Newton-Raphson's method to converge upon the solution.

Configurations with a wire (or several wires) as the corona electrode will be considered in this paper as it is often the case in most applications. The corona effect is assumed to take place simultaneously all around the surface of the wire so that the Kaptzov condition is automatically satisfied by fixing the electric flux F on this electrode through a suitable Dirichlet condition on Γ_d :

$$F = \varepsilon_0 S E_0 \qquad (9)$$

S is the perimeter of the corona electrode and E_0 is the onset value of the electric field derived from the Peek's formula [1]:

$$E_0 = 31\,(\text{kV/cm})\ m\,\delta \left(1 + \frac{0.308}{\sqrt{\delta a}}\right) \qquad (6)$$

where a is the electrode radius in cm, δ is the relative air density factor and m is the roughness factor. The V–I characteristic of the investigated configuration is then obtained by increasing step by step the space charge density, assumed homogeneous, injected on the surface of the corona electrode. Then as an additional advantage there is no need for an outer loop in order to satisfy the Kaptzov condition as it is the case with the electric scalar potential formulation. The voltage value V is obtained in a "variationally correct" way [5].

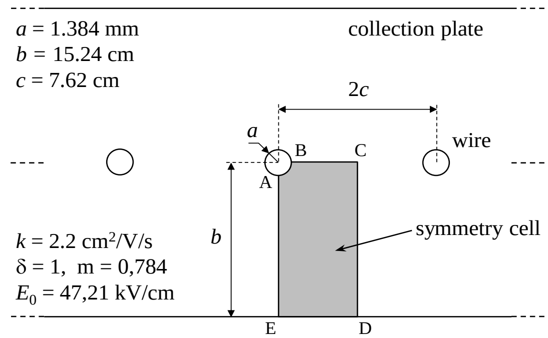

Fig. 1. Wire-duct precipitator modeling.

III. Numerical Results

The described algorithm has been applied for the simulation of the wire-duct electrostatic precipitator with infinite number of wires shown in Fig. 1. Positive corona is assumed. All the required physical and geometrical parameters are indicated. Only the symmetry cell is discretized. The essential (Dirichlet) condition is applied on boundaries AE ($P = F$) and BCD ($P = 0$). The Kaptzov condition specifies the value E_0 on arc AB (the same as for the experimental data of [6]). In Fig. 2 the accuracy of our results is compared to some others deduced from various methods as described in [1] and [6].

Further explanation will be given in the full paper.

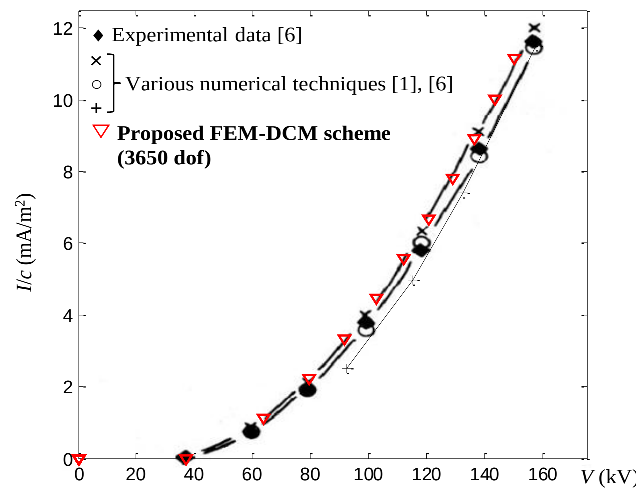

Fig. 2. Validation of our algorithm with published data.

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