

# Level Set-based Topology Optimization for the Design of Light-Trapping Structures

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**Abstract**—This paper discusses a topology optimization method for the design of light-trapping structures that can enhance the absorption capability of thin film solar cells. Light-trapping techniques extend the length of the light path in the active layer and thereby enhance solar cell efficiency. The level set-based topology optimization method is constructed for the structural design of a light-scattering layer attached to the top of the active layer, sandwiched between cladding layers. The optimization problem is formulated to maximize the light absorption coefficient of the solar cells. The numerical result demonstrates that the optimization successfully found an appropriate configuration for a light-scattering layer that enhances the light absorption coefficient at the desired wavelength.

**Index Terms**—Lighting control, Solar energy, Design optimization, Finite element method.

## I. INTRODUCTION

Thin film solar cells are advantageous for their size and material cost, but they provide relatively low efficiency because their active layer is much thinner than the wavelength of light. The efficiency of such solar cells can be improved by using a light-trapping technique in which the length of the light path is extended. Several light-trapping structures have been reported, based on triangular or cubic structures [1], and a dielectric-metal grating [2], but the design of these structures was mainly based on intuition. To develop light-trapping structures that maximize the efficiency of solar cells, a systematic design approach, such as a topology optimization method [3], may be most effective.

Wang et al. [4] presented a systematic design approach using genetic algorithms (GAs) for the cross-section design of a scattering layer that maximizes the light absorption coefficient in the active layer. Their method uses a rigorous coupled-wave analysis method to compute the light absorption coefficient, and GAs for updating design variables. However, meta-heuristic approaches such as GAs are generally not suitable for topology optimization, since the number of design variables is usually so large that the optimization becomes too computationally costly. Thus, in this paper, we apply a gradient-based topology optimization approach for the structural design of a light-scattering layer that maximizes the light absorption coefficient. The finite element method (FEM) is applied to solve the electromagnetic wave problem and the light absorption coefficient is computed based on the solution of the electric field. The optimization algorithm uses the adjoint variable method for the sensitivity analysis and the FEM when solving the electromagnetic propagation and adjoint problems. The results of a three-dimensional design problem demonstrate that the proposed method successfully found an appropriate configuration for the light scattering

layer that enhances the light absorption coefficient at the desired wavelength.

## II. FORMULATION OF OPTIMIZATION PROBLEM

### A. Light Scattering Layer Design Problem

The design domain and boundary conditions of the solar cell model are shown in Fig. 1. Incident waves enter the domain from the top, and the side boundaries are set as a perfect electric conductor (PEC) and a perfect magnetic conductor (PMC), under a periodic condition. The governing equation is given by the following wave propagation equation.

$$a_1(\mathbf{E}, \tilde{\mathbf{E}}) + a_2(\mathbf{E}, \tilde{\mathbf{E}}) = l(\mathbf{E}) \quad \text{for } \mathbf{E} \in U, \forall \tilde{\mathbf{E}} \in U \quad (1)$$

where

$$\begin{aligned} a_1(\mathbf{E}, \tilde{\mathbf{E}}) &= \int_D (\nabla \times \tilde{\mathbf{E}}) \cdot (\nabla \times \mathbf{E}) d\Omega - k_0^2 \int_D n^2 \tilde{\mathbf{E}} \cdot \mathbf{E} d\Omega \\ a_2(\mathbf{E}, \tilde{\mathbf{E}}) &= \int_{\Gamma_1 \cup \Gamma_2} (\mathbf{n} \times \tilde{\mathbf{E}}) \cdot (\mathbf{n} \times \mathbf{E}) d\Gamma \\ l(\tilde{\mathbf{E}}) &= 2jk_0^2 \int_{\Gamma_1} \tilde{\mathbf{E}} \cdot \mathbf{E}^i d\Gamma \\ U &= \{\tilde{\mathbf{E}} \in H^1(\Omega)\} \end{aligned} \quad (2)$$

where  $\mathbf{E}$  is the electric field and  $k_0$  is the wave number in a vacuum.  $\mathbf{n}$  is the normal vector and  $\mathbf{E}^i$  is the incident electric field.

### B. Optimization Problem

The aim of the optimization problem is to find the configuration of dielectric material in the scattering layer that maximizes the light absorbing coefficient. Therefore, the objective function is set to maximize the light absorbing coefficient, which can be computed using the Poynting vector  $S$ , based on the results of the FEM analysis, as follows:

$$\inf_{\phi} F(\phi) = \frac{\int_{\Gamma_2} \mathbf{n} \cdot \text{Re}(S) d\Gamma + \int_{\Gamma_1} \mathbf{n} \cdot \text{Re}(S) d\Gamma}{\int_{\Gamma_1} \mathbf{n} \cdot \text{Re}(S^i) d\Gamma} \quad (3)$$

$$\text{subject to: Wave propagation equation (1)} \quad (4)$$

Boundary conditions

The surface integrals of the Poynting vector  $S$  over  $\Gamma_1$  and  $\Gamma_2$  represent the summation of incident and reflected energy, and transmitted energy, respectively. In the proposed method, the refractive index  $n$  in the scattering layer is represented using the characteristic function  $\chi_{\phi}$ , as follows:

$$n = (n_{\text{die}} - n_{\text{air}}) \chi_{\phi} + n_{\text{air}}, \quad (5)$$

where  $n_{\text{die}}$  and  $n_{\text{air}}$  represent the refractive index of the dielectric material and air, respectively. The characteristic function  $\chi_{\phi}$  is defined using the level set function, which is explained in the next subsection.

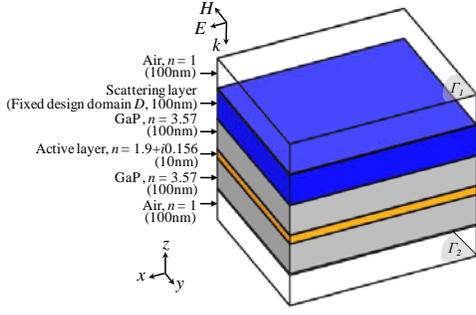


Fig. 1. Design domain and boundary conditions

### C. Level Set-based Topology Optimization

Here we briefly introduce the applied optimization method [5]. Topology optimizations are formulated using a fixed design domain  $D$  that consists of a solid domain  $\Omega$  and a void domain  $D \setminus \Omega$ . In level set-based methods, the solid and void domains and the structural boundaries  $\partial\Omega$  are expressed using the iso-surface of the level set function  $\phi$ , as follows:

$$\begin{cases} 0 < \phi(x) \leq 1 & \text{if } \forall x \in \Omega \setminus \partial\Omega \\ \phi(x) = 0 & \text{if } \forall x \in \partial\Omega \\ -1 \leq \phi(x) < 0 & \text{if } \forall x \in D \setminus \Omega \end{cases} \quad (6)$$

The characteristic function that represents the material distribution is described as follows using the level set function defined above:

$$\chi_\phi(\phi) = \begin{cases} 1 & \text{if } \phi \geq 0 \\ 0 & \text{if } \phi < 0 \end{cases} \quad (7)$$

In the employed level set-based topology optimization, the level set function is updated by solving the following time evolutionary equation.

$$\begin{cases} \frac{\partial \phi}{\partial t} = -K(\phi)(\bar{F}' - \tau \nabla^2 \phi) \\ \frac{\partial \phi}{\partial n} = 0 & \text{on } \partial D \setminus \partial D_N \\ \phi = 0 & \text{on } \partial D_N \end{cases} \quad (8)$$

where  $K$  is coefficient of proportionality,  $\bar{F}$  is Lagrangian of optimization problem,  $\tau$  is regularization coefficient. Details of this method are provided in the reference [5].

### III. NUMERICAL EXAMPLE

Here, we apply our topology optimization method to a three-dimensional light-scattering layer design problem. The dielectric material used has a refractive index of  $n = 3.57$ , assuming that this material is similar to GaP. The refractive indices used for the active layer and cladding material are set as  $n = 1.9 + i0.156i$ , and  $n = 3.57$ , respectively. A circular disc shape with a volume fraction of 50% of the design domain (Fig. 3(a)) is used as the initial configuration. The optimization

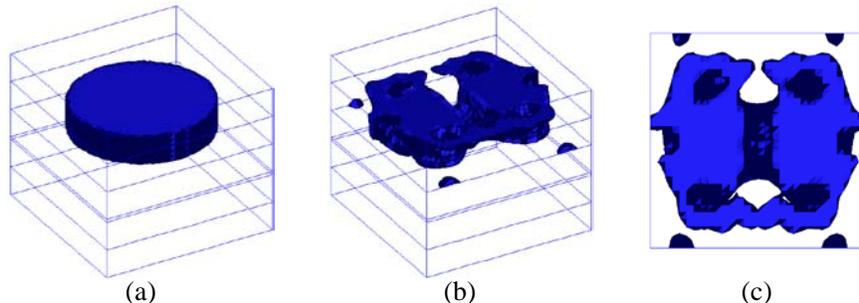


Fig. 3. (a) Initial configuration; (b) obtained configuration; and (c) top view of obtained configuration

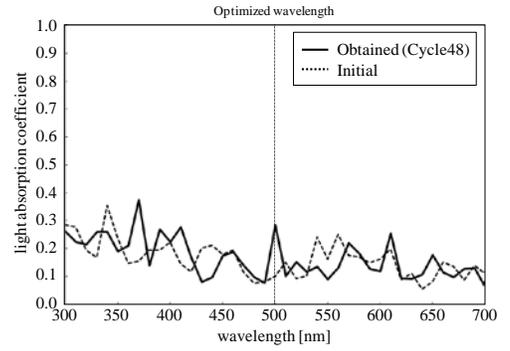


Fig. 2. Wavelength vs. light absorbing coefficient for initial and optimized configurations

wavelength is set as 500 nm. Fig. 3(b) shows the obtained configuration of dielectric material in the light scattering layer. Fig. 2 shows a plot of wavelength versus the light absorbing coefficient, from 300 nm to 700 nm, for the initial and optimized configurations. The absorption coefficient at optimization wavelength was increased to approximately 0.28.

### IV. CONCLUSION

In this study, a level set-based topology optimization method was applied to the design of a light-scattering layer for a solar cell application. The optimization method was formulated to maximize the light absorbing coefficient. The numerical example for a three-dimensional design problem demonstrated that the presented method successfully found a dielectric configuration for the light-scattering layer that maximizes the light absorbing coefficient at the desired wave length. It should be possible to extend the presented method to problems that include multiple wavelengths, and to broadband design problems.

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