Vector Design Optimizations Using an Improved Cross-Entropy Method

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***Abstract*—An improved vector cross-entropy method is proposed to solve multi-objective design optimizations of inverse problems. To balance exploitation and exploration searches, the entire evolutionary process is recast to two phases: diversification and intensification phases. Different parameter updating mechanisms of the probability density function （*pdf*） are designed for each phase. To enhance the diversity of populations, several different *pdf*s are evolved synchronously; to guarantee uniform distribution of the searched Pareto solutions, an *elite* projection mechanism is proposed. Mathematical function and an inverse problem are used to testify the effectiveness and efficiency of the proposed method. The results show that the proposed method can obtain well distributed Pareto solutions using a less iterative number.**

***Index Terms*—Pareto optimal, heuristic algorithms, linear antenna arrays.**

I. An Improved Cross-Entropy Method

A. *Cross-Entropy method*

Based on estimating the probabilities of rare events, the Cross-Entropy (CE) method is firstly proposed as an effective single objective optimizer [1]. Due to the simplicity in both mathematical concepts and numerical implementations, the CE method has also been extended to multi-objective optimizations [2]-[3]. However, a large number of sampling points are still needed to precisely evolve the parameters of *pdfs*. To speed up the convergence speed and obtain better distribution of the Pareto front, some improvements are proposed.

B. *The Diversification Phase*

To balance the exploitation and exploration searches, the whole iterative procedure is divided into two phases: diversification and intensification phases. Diversification phase emphasizes on guaranteeing the diversity of sampling points and intensification phase aims on ensuring a fast convergence. Therefore, different parameter evolutionary mechanisms are proposed for *pdfs* in each phase. The algorithm starts from a diversification phase to uniformly explore the whole feasible space; because of simplicity to implement, the normal distribution function $N(\mu,\sigma^2)$, with its mean μ and standard deviation σ, is selected as the *pdf*. To enhance the diversity of sampling points, several different *pdf*s are evolved synchronously. In the diversification phase, the parameters of *pdfs* evolve using:

$$\mu_j(t+1)=S_j(t) \tag{1}$$

$$\sigma_j(t+1)=std(x_j^{\rho N}(t)) \tag{2}$$

$$\sigma_j(t+1)=\alpha\sigma_j(t+1)+(1-\alpha)\sigma_j(t) \tag{3}$$

$$\alpha=\sigma_0-\sigma_0(1-1/t)^q \tag{4}$$

where; j is the index for the j^{th} *pdf*; $S(t)$ is a set of fixed length, comprised of members of $mean(x_j^{\rho N}(t))$ and external archive A based on tournaments; ρ is the *elite* rate of one sampling; q is an attenuation factor, which is decided by the designer.

C. *The Intensification Phase*

The intensification phase is designed to efficiently and precisely locate the Pareto solutions. In the intensification phase, the parameters of *pdfs* are evolved as follows:

$$\mu(t+1)=A(t)_{elite-related} \tag{5}$$

$$\sigma_j(t+1)=std(x_j^{\rho N}(t)) \tag{6}$$

$$\sigma_j(t+1)=\beta\sigma_j(t+1)+(1-\beta)\sigma_j(t) \tag{7}$$

where; $A(t)_{elite-related}$ is the corresponding *elite* component in the external archive; ρ is the *elite* rate of one sampling; β is a smoothing parameter, which is in the region of 0.6~0.8.

D. *Elite Projection Mechanism*

In order to guarantee the uniform distribution of the searched Pareto solutions, an *elite* projection mechanism is proposed as: Project the *elite* individuals to the utopia sub-domains; classify the projected points into the corresponding sub-domains and calculate the distance between the projection points and the centers of its corresponding sub-domains. Compared with multi-objective normal boundary intersection method, only *elite* individuals are projected onto the utopia plane in the proposed CE, which decreases the computational burden effectively.

E. *Tuning of the Initial Parameters of pdfs*

For the proposed CE method, two parameters, the initial location of parameters of *pdfs* and the sampling size, should be tuned carefully. If the initial values of these parameters are too close to the boundary of the searching space, a lot of sampling points will exceed the boundary; and if the initial parameters are too close to each other, the exploration ability is degraded. As a result, a large amounts of sampling points are needed to modify the searching direction for the two cases. Since the initial parameters are selected randomly in existing CE methods, the aforementioned two ultimate cases may occur. To address this issue, a novel method to determine the initial parameters of the *pdfs* is proposed as: generate N_0 sampling points randomly; and classify the sampling points into the corresponding sub-domains according to the projective distance; after sorting the sampling points based on sub-domains and dividing them into some fix length matrix, a series of μ_0 and σ_0 are finally calculated from each matrix.

F. Algorithm description

To facilitate the understanding of the proposed method, the details about its iterative process are summarized as follows:

Step 1 Calculate the utopia plane.

Step 2 Divide the utopia plane into sub-domains.

Step 3 Calculate the initial parameters of *pdfs*.

Step4 Define N_D the sampling number of the diversification phase; N_I the sampling number of the intensification phase; N_P the population size for one *pdf* family sampling; *A* the external archive; *E* the set of best solutions in one iteration. Initialize iterative number *t*=0; Generate the initial population *P* using the normal distribution function with μ_0 and σ_0.

Step 5 Calculate the function values of population *P* and compute the *E* using a fast non-dominated sorting approach [4]. Project the individuals of *E* into the utopia sub-domains. Classify the projected points into the corresponding sub-domains and calculate the distance between the projection points and the centers of its corresponding sub-domains. Update external archive *A* with individuals of the minimum distance and the lowest rank in the same sub-domains both in set *A* and *E*.

Step 6 If $t \le N_D$, μ_t is updated based on the tournament mechanism from set *A* and μ_t^* updated from *elite* points of each sampling population; σ_t is updated based on the *elite* points of each sampling population, and then σ_t is smoothed using a dynamic function. If $N_D < t \le (N_D + N_I)$, μ_t is updated using the external archive *A*; σ_t is updated based on the *elite* points of each sampling population. σ_t is smoothed using a constant function. If $t > (N_D + N_I)$, go to Step 7.

Step 7 Stop the algorithm.

II. NUMERICAL RESULTS

To validate and demonstrate the advantages of the proposed algorithm, a series of test functions (MOP4) and the linear antenna array optimization problem [5] are solved.

The parameters of the proposed algorithm for solving MOP4 are set as: N_P =20, ρ =10%, N_D =10, N_I =20, *q*=50, β =0.7; Fig. 1 gives the searched Pareto front using the proposed method. Table I tabulates the comparisons of the original MOCE [3], NSGA2 and the proposed method.

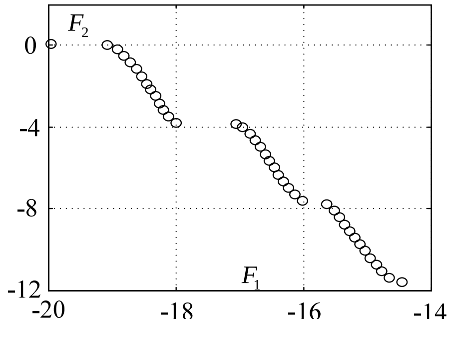

Fig. 1 The searched Pareto front of MOP4 using proposed method

TABLE I
COMPARISON OF PROPOSED ALGORITHM AND OTHER ALGORITHMS

Algorithm	Min F_1	Min F_2	No. of Iterations
Original MOCE	-20.0000	-11.6273	1500000
NSGA2	-19.9974	-11.6273	28000
The proposed	-20.0000	-11.6271	20418

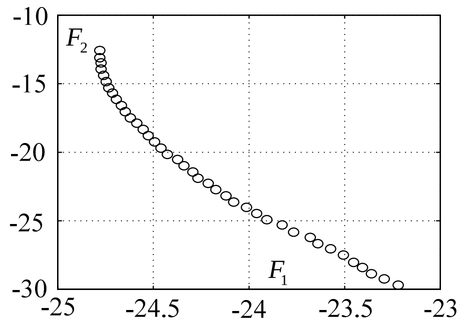

Fig. 2 The searched Pareto Front of the linear antenna array optimization using the proposed method

TABLE II
COMPARISON OF THE FINAL SOLUTIONS AMONG DIFFERENT ALGORITHMS WITH BENCHMARK VALUE OF A UNIFORM ANTENNA ARRAY

Algorithm	No. of non-dominated solution	No. of superior solution	No. of non-Pareto solution
Original MOCE	0	40	0
NBI method	2	38	8
The proposed	0	40	0

The algorithm parameters for solving the linear antenna array optimization are set as: N_P =50, ρ =10%, N_D =15, N_I =20, *q*=50, β =0.7 to update the parameters of *pdf*s using (1)-(7). Fig. 2 presents the searched Pareto front using the proposed method, and Table II gives the comparison on the finial solutions of well designed evolutionary algorithms and the proposed method using the benchmark value of a uniform antenna array. Obviously, the proposed algorithm can find the same qualified solution but using a relative small number of iterations.

III. REFERENCES

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