# A Multi-Layer Finite Element Method Algorithm for Three-Dimensional Magnetic Force Computation

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***Abstract* — This paper presents a novel multi-layer methodology for three-dimensional magnetic force computation. In contrast to conventional methods in which the force is integrated only on single layer, this method calculates the force by integration on several layers of mesh elements around the objects. The program can automatically determine the layer number during computation to accurately address problems with objects touching each other. Finally numerical examples are given to compare the proposed method and the traditional one layer method. The reported results confirm that the superior accuracy of the multi-layer method for force computation.**

***Index Terms* — Finite element method, magnetic fields, multi-layer, three dimension.**

## I. INTRODUCTION

Finite element method (FEM) is a powerful numerical method widely used for electromagnetic field computation. With the advent of computers in terms of processing speed and memory size, 3-dimensional (3-D) FEM is increasingly being used for accurate field computation. Since the numerical errors of these methods are very sensitive to the space discretization around the objects for force computation, very high mesh density is always required to ensure high precision of force computation [1]. However, a very dense mesh inevitably requires a lot of computing time. Methods to reduce mesh descretization density while upholding the high accuracy of magnetic force computation is one of the challenging tasks for researchers.

Maxwell stress tensor and virtual work method are two of the most commonly used methods to analyze the magnetic force of electric devices [2, 3]. Maxwell stress tensor method calculates force based on the electromagnetic field on the surface surrounding the objects. It requires a very fine discretization of space to reduce the numerical errors, which are attributed to themagnetic fields changing sharply in the vicinity of material surfaces. The virtual work method however determines the magnetic force by evaluating the derivation of the co-energy or energy with respect to a virtual displacement. Compared with the Maxwell stress tensor method, the virtual work method is more accurate for the same density of mesh discretization. In most literatures about virtual work method, the force value is computed by integration on one layer of elements around the objects.

In this paper a mesh-insensitive force computation method of 3-D FEM is presented. The force value is obtained by integration on several layers of mesh elements around the objects on which the force is being computed. The number of layers can be determined automatically during computation and the force computation on objects touching each other can also be evaluated accurately. Numerical examples of magnetic force computation are carried out to showcase the effectiveness of the proposed method. The computed results are compared with those obtained using traditional one layer method to showcase the improvements of the proposed algorithm.

## II. METHODS

To illustrate the proposed method, one simple model is shown in Fig. 1, in which the force on Object 1 is to be computed. In accordance to the novel method, Object 2 is created surrounding Object 1. It is a virtual object including Object 1 with one layer of additional elements. The material property of that one layer of elements is the same as that of the background. Due to reciprocity, the force values on Objects 1 and 2 are the same. However, the computed values are not exactly the same because of numerical error. In particular, the computed force on Object 1 is likely to be less accurate that those of Object 2 because of sharp changes in the magnetic field near the former and hence the numerical error is usually larger on Object 1. If many such layers are created and by averaging the computed force values of these multi-layers, the sensitivity to mesh quality is then highly reduced and the accuracy can be greatly improved.

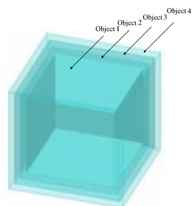


Fig. 1. Virtual objects are constructed in 3-D FEM model for force computation.

When dealing with problems in which the object in issue touches another object, a shell element is used for the force integration along the touched surface [4]. A shell element which is a triangular prism with its height δ approaches zero is used for the force integration.

The contribution of one shell element to the nodal force can be obtained by the derivative of its coenergy with respect to displacement s:

$$F = \left.\frac{\partial W_{co}}{\partial s}\right|_{i=\text{constant}} = \frac{\mu}{2}\sum_{i=1}^{9}\sum_{j=1}^{9} l_i \boldsymbol{H}_i \left(\frac{\partial m_{ij}}{\partial s}\right) l_j \boldsymbol{H}_j \tag{1}$$

where, $$m_{ij} = \int_{R_e} \boldsymbol{w}_i^T \cdot \boldsymbol{w}_j \mathrm{d}R = \int_{\hat{R}_e} \hat{\boldsymbol{w}}_i^T J^{-T} \cdot J^{-1} \hat{\boldsymbol{w}}_j |J| \mathrm{d}\hat{R} \quad (2)$$

where $\boldsymbol{w}$ is the shape function of the edge element. The details of the formulation can be found in [2, 3].

## III. Numerical Experiments

To evaluate the effectiveness of the proposed method, the team workshop problem 23 is used to testify the effectiveness of the method. This team workshop problem aims to calculate the magnetic force between a PM and a coil fed with constant current [5]. The magnet is cylindrical and the coil has 280 turns of wire wound on a nonmagnetic form. The dimension of the geometry is given in the team workshop problem. There are 6 layers of virtual objects which are automatically created surrounding the magnet. In this project, the axial force between the magnet and the coil carrying a dc current of 50 mA at different distances is computed. With the distance between the magnet and coil fixed at a distance of d = 0.514 mm in this study, the axial force between the magnet and the coil with different current values is computed.

Tables I and II give the calculation results and comparison results. With this proposed method, the error is obviously reduced compared to that with the conventional method (single layer method).

TABLE I Force Solutions

| Layer No. | Axial force (mN) |
|---|---|
| 1 | 0.95414 |
| 2 | 0.93907 |
| 3 | 0.92492 |
| 4 | 0.92560 |
| 5 | 0.92834 |
| 6 | 0.92452 |
| 7 | 0.93701 |
| Average (2-6) | 0.92894 |

TABLE II Comparisons of Force with Different Methods

| Items | Conventional method | Multi layers method |
|---|---|---|
| Axial displacement (mm) | 0.514 | 0.514 |
| Measured results (mN) | 0.91233 | 0.91233 |
| Calculated results (mN) | 0.95414 | 0.92894 |
| Error (%) | 4.5827% | 1.8206% |

The method is then applied to compute the magnetic force distribution. A simple example with two rectangular PM objects attracting each other is used to verify the numerical method against its analytical solution. In the example each PM object is 40mm×20mm×10mm and $B_r$=1.1 tesla, $\mu$=$\mu_0$. They are separated by 10mm from each other along the *y* axis. Fig. 2 shows the virtual layers for force measurement. Table III shows the computed forces at different layers with a gap of 10 mm (along the z axis) between the two objects with the proposed method. Table IV shows the comparisons of force calculation with the proposed method, conventional method and analytical method. It shows that with the proposed method, the numerical error is reduced further by 1.16% compared with those obtained using the conventional method. Table V shows the computed forces at different layers with a gap of 9 mm between the two objects using the proposed method. Table VI shows the comparisons of force calculation with the proposed method, conventional method and analytical method. It shows that with the proposed method, the numerical error is reduced further by 1.9 % when compared with those using the conventional method.

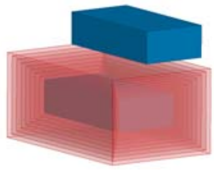

Fig. 2. Virtual layers for force measurement.

TABLE III Force Solutions of Case II (gap=10 mm)

| Layer No. | $F_y$ | $F_z$ | $F$ |
|---|---|---|---|
| 1 | 21.2678 | 20.2386 | 29.35848 |
| 2 | 21.3792 | 20.6206 | 29.70319 |
| 3 | 21.3751 | 20.6518 | 29.72191 |
| 4 | 21.3783 | 20.6503 | 29.72317 |
| 5 | 21.3751 | 20.6490 | 29.71996 |
| 6 | 21.3761 | 20.6503 | 29.72158 |
| 7 | 21.3777 | 20.6527 | 29.7244 |
| 8 | 21.3792 | 20.6507 | 29.72409 |
| 9 | 21.3569 | 20.4271 | 29.55306 |
| Average (2-8) | 21.37724 | 20.64649 | 29.71976 |

TABLE IV Comparisons of Force with Different Methods (gap=10 mm)

| Items | Analytical results (N) | Conventional Method (N) | | Proposed Method | |
|---|---|---|---|---|---|
| | | Value (N) | Error (%) | Value (N) | Error (%) |
| $F_y$ | 23.354706 | 21.2678 | 8.936 | 21.37724 | 8.467 |
| $F_z$ | 20.62964 | 20.2386 | 1.896 | 20.64649 | 0.0817 |
| $F$ | 31.16126 | 29.35848 | 5.785 | 29.71976 | 4.626 |

TABLE V Force Solutions of Case II (gap=9 mm)

| Layer No. | $F_y$ | $F_z$ | $F$ |
|---|---|---|---|
| 1 | 23.6726 | 22.0501 | 32.35118 |
| 2 | 24.2296 | 22.3817 | 32.98506 |
| 3 | 24.2421 | 22.3863 | 32.99736 |
| 4 | 24.2460 | 22.3989 | 33.00878 |
| 5 | 24.2450 | 22.4006 | 33.00919 |
| 6 | 24.2457 | 22.4042 | 33.01215 |
| 7 | 24.2483 | 22.4052 | 33.01474 |
| 8 | 24.2536 | 22.3960 | 33.01239 |
| 9 | 24.2190 | 22.0669 | 32.76443 |
| Average (2-8) | 24.24433 | 22.39613 | 33.00567 |

TABLE VI Comparisons of Force with Different Methods (gap=9 mm)

| Items | Analytical results (N) | Conventional Method (N) | | Proposed Method | |
|---|---|---|---|---|---|
| | | Value (N) | Error (%) | Value (N) | Error (%) |
| $F_y$ | 26.639069 | 23.6726 | 11.136 | 24.24433 | 8.99 |
| $F_z$ | 22.376133 | 22.0501 | 1.457 | 22.39613 | 0.0893 |
| $F$ | 34.78982 | 32.35118 | 7.01 | 33.00567 | 5.128 |

## References


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