Coupling of a method of moments adapted to planar circuit and volumic methods

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***Abstract*—Hybridization between a method of moments called WCIP (Wave Concept Iterative Procedure) and volumic methods like the FDTLM (Frequency Domain Transmission Line Method), the FEM (Finite Element Method) and the HDG (Hybridizable Discontinuous Galerkin) method is presented in 2d in this paper. The considered problem is the Helmholtz equation in the frequency domain. Two test cases are provided to validate the proposed hybridization principle.**

***Index Terms*—Electromagnetic modeling, microwave propagation, finite element methods.**

I. Introduction

Our work is part of circuits modeling in high frequencies. In particular, we aim at studying electromagnetic susceptibility of planar circuits. The WCIP [1] is a specific method adapted to microwaves planar circuits study. Nevertheless, this latter cannot characterize circuits with dielectric inhomogeneities [2]. In this context, we are concerned in this study with the hybridization between different numerical methods in the frequency domain. The WCIP has been hybridized with a finite element method (FEM-Q_1), a hybridized discontinuous Galerkin method (HDG) [3] and a method based on transmission lines theory (FDTLM) [4]. Two 2d validation examples are dealt with in this short paper to validate the resulting hybrid methods. TM and TE cases have been studied, but only TE results are presented here.

II. Hybridization principle

The computational domain is separated into two parts to simplify the approach as it is shown in figure 1. Domain 1 is tackled with the WCIP whereas domain 2 is addressed with another method; the connection is achieved at the interface. The linear system to be solved is:

$$\left(\mathrm{I_d} - \begin{pmatrix} \mathrm{S_1^W} & 0 \\ 0 & \mathrm{S_2^F} \end{pmatrix} \mathrm{S}\right) \begin{pmatrix} \mathrm{B_1} \\ \mathrm{B_2} \end{pmatrix} = \begin{pmatrix} \mathrm{B_0} \\ 0 \end{pmatrix}, \tag{1}$$

where $\mathrm{I_d}$ is the identity matrix, $\mathrm{S_1^W}$ the discretization of the WCIP operator, defined by:

$$\mathrm{S_1^W} = \mathrm{FMT}^{-1}\Gamma_1\mathrm{FMT} \tag{2}$$

with Γ_1 the diagonal matrix composed of modal diffraction coefficients, FMT the discretization of a fast modal transform, $\mathrm{S_2^F}$ the operator discretization of domain 2, S the transmission operator between both domains, $\mathrm{B_1}$ and $\mathrm{B_2}$ incident waves on the interface (Σ) (see figure 1) and $\mathrm{B_0}$ the source.

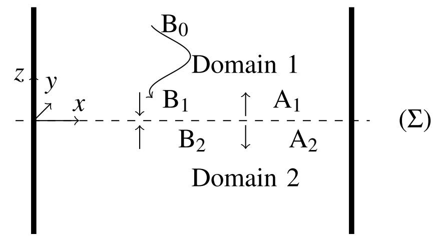
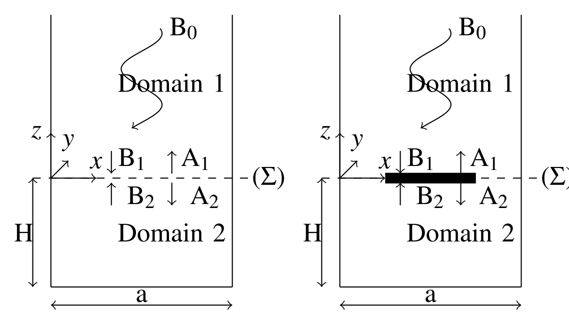


Figure 1: Representation of the studied case, separation between both domains according to the interface.

III. Numerical results

A. *Diffraction of a guided mode on a perfect sheet*

The example of figure 2a is taken into account with $\mathrm{H} = 1,27\mathrm{cm}$ and $\mathrm{a} = 1,27\mathrm{cm}$ at 16 GHz. Analytic solution for electric and magnetic fields being known, relative discretization error in $\mathrm{L_2}$-norm is evaluated (see figure 3). FEM is implemented with quadrangular elements (FEM-Q_1) and HDG with triangular elements (HDG-P_1). The HDG-P_1 method provides better results as far as relative error is concerned, and the three methods converge in h^2, h denoting the mesh step.



(a) Waveguide (b) Microstrip line

Figure 2: Examples

A comparison between hybrid methods using HDG-P_0, HDG-P_1 and HDG-P_2 [3] in domain 2 was also performed for E-field (see table I). This comparison shows that convergence order is 1 with HDG-P_0, 2 with HDG-P_1 and also 2 with HDG-P_2 because WCIP limits convergence order, but relative error is improved with HDG-P_2.

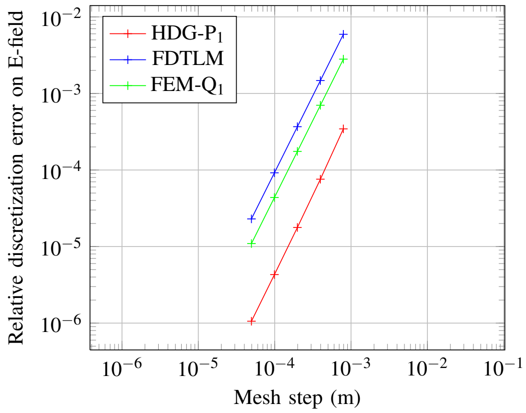


Figure 3: Relative discretization error in L_2-norm for E field with TE_1 in excitation.

Table I: Relative discretization error in L_2-norm for hybridization with HDG-P_0, HDG-P_1 and HDG-P_2.

Mesh size	HDG-P_0	HDG-P_1	HDG-P_2
1	$6.48.10^{-2}$	$3.45.10^{-4}$	$1.63.10^{-4}$
1/2	$3.69.10^{-2}$	$7.58.10^{-5}$	$3.70.10^{-5}$
1/4	$1.97.10^{-2}$	$1.78.10^{-5}$	$8.79.10^{-6}$
1/8	$1.02.10^{-2}$	$4.30.10^{-6}$	$2.14.10^{-6}$
1/16	$5.20.10^{-3}$	$1.06.10^{-6}$	$5.28.10^{-7}$

B. *Diffraction of a guided mode on a microstrip line*

A microstrip line is inserted on the surface (Σ) (see figure 2b). It is centered and the metal proportion compared to air is 50%. We inject the TE_1 mode on the microstrip and we calculate the relative error on the E-field and the J-current. In this case, we do not know analytic solution and therefore, the chosen reference is the solution obtained with the WCIP alone, meshing the domain with $N = 2^{15}$ where N is the number of segments on (Σ). Relative errors on electric field and current are respectively summarized in tables II and IV and convergence orders are given in tables III and V.

Table II: Relative discretization error in L_2-norm on E-field.

Mesh size	FEM-Q_1	HDG-P_1	FDTLM
1	$2.34.10^{-2}$	$2.91.10^{-2}$	$2.65.10^{-2}$
1/2	$1.34.10^{-2}$	$1.58.10^{-2}$	$1.47.10^{-2}$
1/4	$7.24.10^{-3}$	$8.23.10^{-3}$	$7.72.10^{-3}$
1/8	$3.86.10^{-3}$	$4.31.10^{-3}$	$4.06.10^{-3}$
1/16	$2.03.10^{-3}$	$2.23.10^{-3}$	$2.11.10^{-3}$

Table III: Convergence orders on E-field.

Mesh size	FEM-Q_1	HDG-P_1	FDTLM
1	-	-	-
1/2	0.8025	0.8827	0.8557
1/4	0.8933	0.9373	0.9237
1/8	0.9059	0.9350	0.9262
1/16	0.9282	0.9507	0.9440

We notice that convergence orders are respectively 1 and 0.5 for E-field and J-current (order reduction coming from the discontinuity between metal and dielectric) in TE case whatever method used in domain 2, with very close relative discretization errors between hybrid methods.

Table IV: Relative discretization error in L_2-norm on J-current.

Mesh size	FEM-Q_1	HDG-P_1	FDTLM
1	$2.77.10^{-2}$	$2.76.10^{-2}$	$2.76.10^{-2}$
1/2	$1.95.10^{-2}$	$1.95.10^{-2}$	$1.95.10^{-2}$
1/4	$1.37.10^{-2}$	$1.37.10^{-2}$	$1.37.10^{-2}$
1/8	$9.65.10^{-3}$	$9.59.10^{-3}$	$9.61.10^{-3}$
1/16	$6.76.10^{-3}$	$6.70.10^{-3}$	$6.72.10^{-3}$

Table V: Convergence orders on J-current.

Mesh size	FEM-Q_1	HDG-P_1	FDTLM
1	-	-	-
1/2	0.5037	0.5048	0.5044
1/4	0.5064	0.5080	0.5073
1/8	0.5097	0.5121	0.5111
1/16	0.5139	0.5174	0.5160

IV. Conclusion

This work enables us to check hybridization principle between the WCIP and other volumic methods. A convergence order of 2 has been emphasized in a canonical case whatever the hybrid method implemented (FEM-Q_1, HDG or FDTLM) and using HDG-P_2 does not improve convergence order. The insertion of a microstrip line between both domains is also very relevant, because the 3 methods provide similar results, namely a convergence order of 1 for E-field and an order of 0.5 for electric current for a TE_1 mode in excitation. Consequently, inhomogeneous substrates, not dealt with the WCIP alone, will be studied with these hybrid methods keeping the advantages of surface conditions of the WCIP. This work is promising for the hybridization in 3d.

V. Acknowledgments

The authors acknowledge support from the Defence Procurement Agency (DGA).

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