Initial Value Problem Formulation of 3D Time Domain Boundary Element Method

H. Kawaguchi[1], S. Itasaka[1] and T. Weiland[2]

[1] Muroran Institute of Technology, 27-1, Mizumoto-cho, Muroran, 050-8585, Japan

[2] Technische Universitaet Darmstadt, Schlossgartenstrasse 8, D-64289 Darmstadt, Germany

E-mail : kawa@mmm.muroran-it.ac.jp

***Abstract*—A time domain boundary element method (TDBEM) gives us another possibility of time domain microwave simulations in addition to a finite difference time domain (FDTD) method. However, it is known that the TDBEM often shows us instability owing to interior resonance. To avoid the interior resonance caused by an open boundary problem formulation, an initial value problem formulation of 3D TDBEM presented in this paper.**

***Index Terms*—Microwave propagation, Particle accelerators, Moment methods, Numerical simulation**

I. Introduction

A time domain boundary element method (TDBEM) gives us another possibility of time domain simulations of microwave phenomena [1],[2] in addition to a finite difference time domain (FDTD) method. The TDBEM has advantages in open boundary problems, treatments of slightly curved boundary objects, coupled problems with charged particle, etc. compared with the FDTD method. In particular, the coupled problem with charged particles such as analysis of wake fields in a particle accelerator is one of the most suitable applications of the TDBEM owing to surface meshing. However, a treatment of infinite length structure of the particle accelerator is a very difficult subject in the TDBEM, therefore, a numerical model of a finite length accelerator tube with thin thickness is used mainly in conventional works [2]. Then, the TDBEM was often numerically unstable in a long range simulation owing to the thin thickness structure of the numerical model. To improve this problem of a numerical stability, an initial value problem formulation of the TDBEM was presented for axis-symmetric two-dimensional problems [3]. In general, two-dimensional TDBEM can be used in only limited applications. In this paper, the initial value problem formulation of the TDBEM is extended to three-dimensional cases, and it is shown that the numerical stability is improved.

II. Time Domain EFIE/MFIE and TDBEM

It is known that time domain electromagnetic fields, $\mathbf{E}(t,\mathbf{x})$, $\mathbf{B}(t,\mathbf{x})$, in a domain V can be expressed using electromagnetic fields on the domain surface $S=\partial V$ in the following surface integral equation forms [3];

$$\begin{aligned}\mathbf{E}(t,\mathbf{x}) = \mathbf{E}_{ext}(t,\mathbf{x}) &+ \frac{1}{4\pi}\int_S \left\{ \frac{\mathbf{n}'\times\dot{\mathbf{B}}(t',\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} \right\} dS' \\ &+ \frac{1}{4\pi}\int_S \left\{ -\left(\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t} \right)\left(\mathbf{E}(t',\mathbf{x}')\cdot\mathbf{n}'\right) \right\} dS' \\ &+ \frac{1}{4\pi}\int_S \left\{ -\left(\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t} \right)\times\left(\mathbf{E}(t',\mathbf{x}')\times\mathbf{n}'\right) \right\} dS', \end{aligned} \tag{1}$$

$$\begin{aligned}\mathbf{B}(t,\mathbf{x}) = \mathbf{B}_{ext}(t,\mathbf{x}) &+ \frac{1}{4\pi}\int_S \left\{ \frac{\dot{\mathbf{E}}(t',\mathbf{x}')\times\mathbf{n}'}{|\mathbf{x}-\mathbf{x}'|} \right\} dS', \\ &+ \frac{1}{4\pi}\int_S \left\{ \left(\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t} \right)\times\left(\mathbf{n}'\times\mathbf{B}(t',\mathbf{x}')\right) \right\} dS' \\ &+ \frac{1}{4\pi}\int_S \left\{ -\left(\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t} \right)\left(\mathbf{n}'\cdot\mathbf{B}(t',\mathbf{x}')\right) \right\} dS' \end{aligned} \tag{2}$$

which are the time domain version of well-known frequency domain expressions of the EFIE and MFIE [4]. $\mathbf{E}_{\mathrm{ext}}(t,\mathbf{x})$ and $\mathbf{B}_{\mathrm{ext}}(t,\mathbf{x})$ are externally applied electric and magnetic fields, respectively, the retarded time t' is defined by $t'=t-|\mathbf{x}-\mathbf{x}'|/c$, c is the velocity of the light and $\mathbf{n}$ is a unit normal vector on the surface. To discretize Eqs.(1) and (2) in time and surface, we obtain a matrix equation containing many matrices (Fig.1), which should be solved as the 3D TDBEM. Unknown vectors in the matrix equation are two components of a surface current and charge densities $\mathbf{K}$, σ, which correspond to $\mathbf{n}\times\mathbf{B}$ and $\mathbf{n}\cdot\mathbf{E}$, respectively. Owing to the retarded time property of Eqs.(1) and (2), unknowns at different time steps are independent each other. Then, if we assume that the boundary S is a perfectly electric conductor (PEC) throughout, (2) results in the following very simple form;

$$\begin{aligned}\mathbf{B}(t,\mathbf{x}) = \mathbf{B}_{ext}(t,\mathbf{x}) \\ + \frac{1}{4\pi}\int_S \left\{ \left(\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t} \right)\times\left(\mathbf{n}'\times\mathbf{B}(t',\mathbf{x}')\right) \right\} dS'. \end{aligned} \tag{2'}$$

III. Treatment of Infinite Length Structure of Particle Accelerator in TDBEM

In derivation of the integral equations (1) and (2), it is assumed that there are no surface current and charge density at the initial time. In addition, the surface integrals in Eqs.(1) and (2) are carried out for entire domain boundary. To satisfy these conditions and realize the simulation of wake fields by a finite length numerical model, an open boundary problem formulation using a numerical model of a torus topology (Fig.2(a)) was used in conventional works. Then, it was assumed that the electron beam was located at sufficiently far upstream distance from the finite length accelerator tube at the

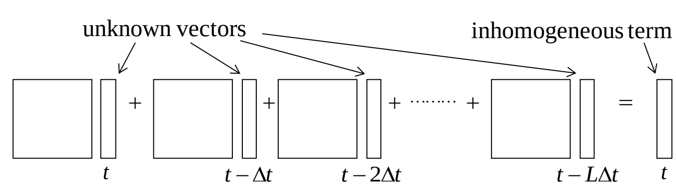

Fig. 1. Structure of matrix equation of TDBEM

initial time to satisfy the conditions of no surface current and charge density on the domain surface. In this formulation, the domain boundary is PEC throughout, therefore, the simplified formulation (2)' can be used. However, the conventional method using the numerical model of Fig.2(a) had some difficulties, a bad matrix property caused by the thin double layer structure of the numerical model, an instability caused by an interior resonance in the long range simulation, a large calculation size owing to the double layer structure of the numerical model.

IV. Initial Value Problem Formulation of 3D TDBEM

To improve these difficulties, which are caused by the open boundary problem formulation (2)', an initial value problem formulation of the TDBEM, which allows us to use the numerical model with a closed domain structure (Fig.2(b)), was proposed for axis-symmetric two-dimensional cases. However, the TDBEM with the axis-symmetric assumption can be used only in limited applications. In this paper, the initial value formulation of the TDBEM is expanded to three-dimensional cases. For the case that there are non-zero surface current and charge densities on the domain surfaace at the initial time, Eqs.(1) and (2) are generalized as follows [3];

$$
\begin{aligned}
\mathbf{E}(t,\mathbf{x}) = {} & \mathbf{E}_{ext}(t,\mathbf{x}) + \frac{1}{4\pi}\int_S \left\{ \frac{\mathbf{n}'\times\dot{\mathbf{B}}(t',\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} \right\} dS' \\
& + \frac{1}{4\pi}\int_S \left\{ -\left(\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t} \right) (\mathbf{E}(t',\mathbf{x}')\cdot\mathbf{n}') \right\} dS' \\
& + \frac{1}{4\pi}\int_S \left\{ -\left(\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t} \right) \times (\mathbf{E}(t',\mathbf{x}')\times\mathbf{n}') \right\} dS' \\
& + \frac{1}{4\pi}\int_{V_0} \left(\left(\dot{\mathbf{E}}(t',\mathbf{x}') + \frac{\partial \mathbf{E}(t',\mathbf{x}')}{\partial r'} \right) \frac{|\mathbf{x}-\mathbf{x}'|}{c} + \mathbf{E}(t',\mathbf{x}') \right) dV',
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
\mathbf{B}(t,\mathbf{x}) = {} & \mathbf{B}_{ext}(t,\mathbf{x}) + \frac{1}{4\pi}\int_S \left\{ \frac{\dot{\mathbf{E}}(t',\mathbf{x}')\times\mathbf{n}'}{|\mathbf{x}-\mathbf{x}'|} \right\} dS' \\
& + \frac{1}{4\pi}\int_S \left\{ \left(\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t} \right) \times (\mathbf{n}'\times\mathbf{B}(t',\mathbf{x}')) \right\} dS' \\
& + \frac{1}{4\pi}\int_S \left\{ -\left(\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t} \right) (\mathbf{n}'\cdot\mathbf{B}(t',\mathbf{x}')) \right\} dS' \\
& + \frac{1}{4\pi}\int_{V_0} \left(\left(\dot{\mathbf{B}}(t',\mathbf{x}') + \frac{\partial \mathbf{B}(t',\mathbf{x}')}{\partial r'} \right) \frac{|\mathbf{x}-\mathbf{x}'|}{c} + \mathbf{B}(t',\mathbf{x}') \right) dV',
\end{aligned}
\tag{4}
$$

Differences in (3) and (4) from the conventional integral equations (1) and (2) are only an existence of the fifth terms of the domain volume integral V_0 at the initial time. The fifth terms in (3) and (4) are superposed on the inhomogeneous term, and the time domain simulation based on (3) and (4) is carried out in the same manner as that of Fig.1.

In Fig.3, a numerical model of the slightly curved accelerator tube is indicated. It is assumed that the electron beam travels on the center axis of the tube. It is found that the FDTD grid generation of this numerical model is not easy, and the 3D TDBEM surface meshing is suitable for this model. To treat a large number of unknowns owing to 3D meshing, MPI parallel processing with four nodes is employed in this calculation. A time domain behavior of the induced surface current (rotational direction) on the observation line in Fig.3 is indicated in Fig.4. For the same mesh size, the conventional method based on the open boundary problem formulation leads to unstable result, on the other hand, the presented method gives us the stable solution.

V. Summary

In this paper, the initial value problem formulation of three-dimensional time domain boundary element method has been presented. Owing to extremely large memory property of 3D TDBEM, MPI parallel processing is employed in the simulation code. The proposed method is applied to the wake field analysis in the slightly curved accelerator tube. It was shown that the numerical stability is improved owing to use of the closed domain numerical model.

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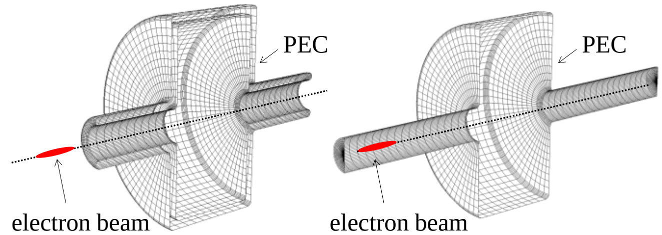

(a) open boundary model (b) closed domain model

Fig.2. Numerical model of a part of particle accelerator tube

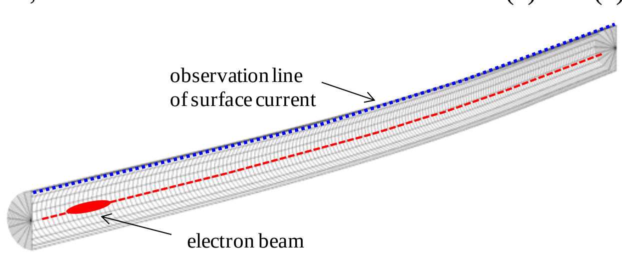

Fig. 3. Numerical model of slightly curved accelerator tube

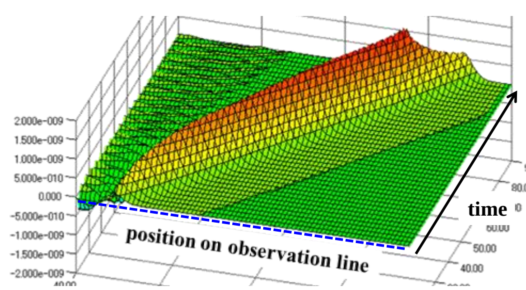

Fig. 4. Time domain behavior of surface current (rotational component)