Linear Representation of Saturation Characteristics Associated with Eddy Currents in Ferromagnetic Materials

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***Abstract*—Based on the linear field theory and the limiting nonlinear theory, respectively, the expressions for the eddy current losses in thick iron plates and thin laminations are obtained. Two new equivalent permeability models are developed to take into account the saturation effects occurring in the corresponding ferromagnetic materials. The proposed approach to the linearization of the field analysis is investigated by comparing the results of the linear analysis using the equivalent permeability with those of the nonlinear finite element analysis.**

***Index Terms*—Eddy currents, magnetic materials, permeability, saturation magnetization.**

I. Introduction

In electrical machines and transformers, ferromagnetic materials applied to form closed magnetic circuits can be categorized into two distinct groups: thick iron slabs (or solid cores) and thin laminations. The solid iron plates are usually easily driven into magnetic saturation due to the small skin depths. For laminated cores, the saturation phenomenon will also inevitably occur, especially in the case of overload operation.

The saturation effects observed in ferromagnetic materials which are subjected to external time-changing magnetic fields are commonly simulated with nonlinear numerical methods using the corresponding magnetization curves [1], [2]. However, in practical applications, the nonlinear numerical analysis is liable to be hampered by the convergence of iteration calculation, the computation time, and the computational stability. Furthermore, when an analytical calculation of the field distributions is preferred, the nonlinear governing equation is difficult to be solved. The purpose of this work is to develop two equivalent permeability models which take into account the saturation behavior in solid iron plates and laminated cores, respectively, and to investigate the feasibility of approximate treatment of the nonlinear eddy current problem by using the linearization formulation.

II. Theory

A. Equivalent Permeability Model for Thick Iron Slabs

In solid-rotor motors, eddy current brakes and couplings, the solid iron parts are subject to the incident magnetic fields. An infinite iron slab of thickness d, as shown in Fig. 1, is considered where a time-varying magnetic field is applied to its upper surface.

1) Linear theory

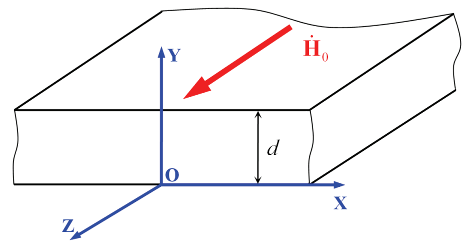

Fig. 1. Ferromagnetic plate with a sinusoidal excitation applied to its surface.

Assuming sinusoidal time variation with the angular frequency ω, the eddy current loss per unit area is found as

$$P_L = \frac{H_o^2}{2\sigma\delta}\frac{\sinh 2d/\delta - \sin 2d/\delta}{\cosh 2d/\delta + \cos 2d/\delta} \tag{1}$$

where δ is the classical skin depth

$$\delta = \sqrt{2/(\mu_{eq}\sigma\omega)}\,, \tag{2}$$

σ is the conductivity of iron material, and μ_{eq} is the equivalent permeability accounting for the saturation characteristics.

2) Limiting nonlinear theory

MacLean [4] and Agarwal [5] suggested that the magnitude of the magnetic flux density may be considered constant if the magnetic field strength is strong enough. According to this theory, the loss per unit area can be given as

$$P_A = \frac{8}{3\pi}\frac{H_o^2}{\sigma\delta_A} \qquad \text{for } \delta_A < d \tag{3}$$

$$P_A = \frac{8}{3\pi}\frac{H_o^2}{\sigma\delta_A}\left[1-\left(1-d^2/\delta_A^2\right)^{3/2}\right] \qquad \text{for } \delta_A \geq d \tag{4}$$

where

$$\delta_A = \sqrt{2H_o/(c_s B_s \sigma\omega)} \tag{5}$$

and H_o is the peak magnetic field strength at the surface of iron plate, B_s being the corresponding magnetic flux density as obtained from the actual steel characteristics. The empirical coefficient c_s is set to 0.75, following Agarwal.

3) Equivalent permeability model

The equivalent permeability μ_{eq} can be obtained by equating the eddy current loss expressions obtained from the above two theories. There are

$$\frac{8}{3\pi\delta_A} = \frac{1}{2\delta}\frac{\sinh 2d/\delta - \sin 2d/\delta}{\cosh 2d/\delta + \cos 2d/\delta} \qquad \text{for } \delta_A < d \tag{6}$$

$$\frac{8}{3\pi\delta_A}\left[1-\left(1-d^2/\delta_A^2\right)^{3/2}\right]=\frac{1}{2\delta}\frac{\sinh 2d/\delta-\sin 2d/\delta}{\cosh 2d/\delta+\cos 2d/\delta}$$
$$\text{for } \delta_A \geq d . \quad (7)$$

B. Equivalent Permeability Model for Thin Laminations

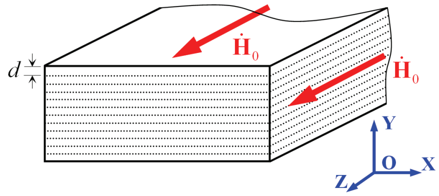

Fig. 2. Laminated iron core subject to a sinusoidal excitation.

For laminated iron cores mounted in electrical equipment, the infinitely long model shown in Fig. 2 is representative.

1) Linear theory

The magnetic field boundary value problem was formulated and solved in our previous work [6]. The eddy current loss per unit area in a lamination is

$$P_L=\frac{H_o^2}{\sigma\delta}\frac{\sinh d/\delta-\sin d/\delta}{\cosh d/\delta+\cos d/\delta} . \quad (8)$$

2) Limiting nonlinear theory

For the thin lamination, the condition

$$\delta_A \geq d/2 \quad (9)$$

is met in most practical applications. In this case, the eddy current loss formula defined by Agarwal is

$$P_A=\frac{16}{3\pi}\frac{H_o^2}{\sigma\delta_A}\left\{1-\left[1-d^2/\left(2\delta_A\right)^2\right]^{3/2}\right\} . \quad (10)$$

3) Equivalent permeability model

The following equation is thus found to calculate the equivalent permeability:

$$\frac{16}{3\pi\delta_A}\left\{1-\left[1-d^2/\left(2\delta_A\right)^2\right]^{3/2}\right\}=\frac{1}{\delta}\frac{\sinh d/\delta-\sin d/\delta}{\cosh d/\delta+\cos d/\delta} . \quad (11)$$

C. Implementation of the Proposed Permeability Model

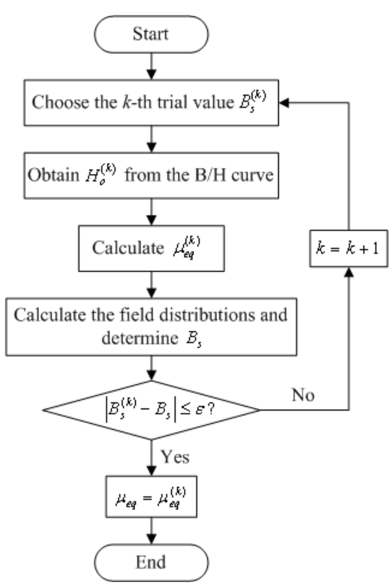

Fig. 3. Process to apply the permeability model.

A flowchart of the application of the presented permeability model incorporated in the linear field analysis is given in Fig. 3. An iterative process is required to determine the equivalent permeability, where the error tolerance ε is typically set equal to 0.1 T.

III. Numerical Verifications

Two analyzed models, a permanent magnet eddy current coupling containing the solid back iron and a laminated core model, are considered to investigate the effectiveness of the proposed permeability formulation. Both the nonlinear finite element analysis combined with the B-H curve of the ferromagnetic material and the linear analysis with the equivalent permeability are performed. Some results are reported in Fig. 4.

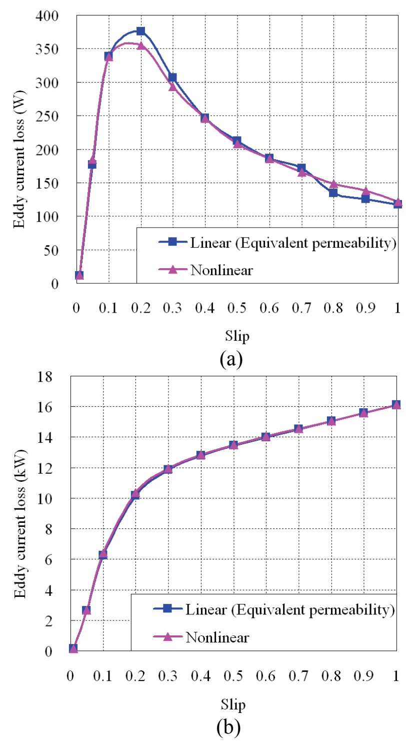

Fig. 4. Eddy current loss versus slip (for the permanent magnet eddy current coupling). (a) Solid iron plate. (b) Copper plate.

References

[1] K. Preis, O. Biro, and I. Ticar, "FEM analysis of eddy current losses in nonlinear laminated iron cores," *IEEE Trans. Magn.,* vol. 41, no. 5, pp. 1412-1415, May 2005.

[2] J. Gyselinck, R. V. Sabariego, and P. Dular, "A nonlinear time-domain homogenization technique for laminated iron cores in three-dimensional finite-element models," *IEEE Trans. Magn.,* vol. 42, no. 4, pp. 763-766, Apr. 2006.

[3] O. Biro and K. Preis, "On the use of the magnetic vector potential in the finite-element analysis of three-dimensional eddy currents," *IEEE Trans. Magn.,* vol. 25, no.4, pp. 3145-3159, July 1989.

[4] W. MacLean, "Theory of strong electromagnetic waves in massive iron," *J. Appl. Phys.,* vol.25, no.10, pp. 1267-1270, 1954.

[5] P. D. Agarwal, "Eddy-current losses in solid and laminated Iron," *Trans. AIEE,* vol. 78, Part I, pp. 169-181, 1959.

[6] J. Wang, H. Y. Lin, Y. K. Huang, and X. K. Sun, "A new formulation of anisotropic equivalent conductivity in laminations," *IEEE Trans. Magn.,* vol. 47, no. 5, pp. 1378-1381, May 2011.