Inclusion of the Model of Rotational Magnetization into Equations of Magnetic Field Distribution

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***Abstract*—The paper deals with the equations of the magnetic field in generator steel sheets. These equations which directly associate components of the magnetic flux density with components of the field intensity are formulated on the basis of the Maxwell equations in the integral form. These equations have been modified by inclusion of the appropriate model of the rotational magnetization. The proposed method which takes the model of rotational magnetization into account was verified experimentally for chosen generator steel sheets.**

***Index Terms*—Electromagnetic modelling, magnetic fields, magnetization processes, soft magnetic materials.**

I. Introduction

The magnetization processes in generator steel sheets have a rotational and very often elliptical character. Due to nonlinear magnetic properties, flux densities in individual points of a steel sheet proceed differently. Therefore, in order to calculate the field distribution, the given generator sheet should be split into elementary segments (Fig. 1). In numerical calculations, an appropriate model of the rotational magnetization must be taken into account in each segment.

The model of rotational magnetization which is used in this application is presented in detail in [4]. The plane of a sample of the given generator sheet is divided into an assumed number of specified directions, similarly as in [1], [3]. To each direction, a certain hysteresis loop - the so-called direction hysteresis - is assigned. The parameters of these direction hystereses are calculated on the basis of such values as the saturation flux density, the residual flux density, and the coercive force of the given generator sheet. The resultant flux density in the elementary segment is the vector sum of the flux densities in the specified directions [2], [3]. It is worth underlining that the direction hystereses differ from the hysteresis loop of the whole sheet sample, and they cannot be determined by any measurement. Determining of the direction hysteresis parameters is described in [4].

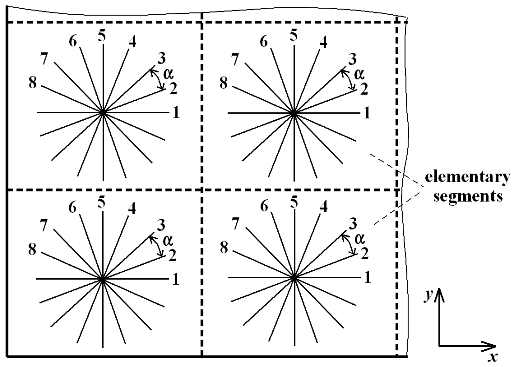

Fig. 1. Part of a generator sheet with elementary segments

The character of the magnetization process in individual elementary segments depends on changes of the field intensity in these segments. Therefore, it is necessary to formulate the equations of the magnetic field distribution in which the field intensities in individual segments are unknown quantities. These equations are derived from the Maxwell equations in the integral form, similarly as in the equivalent reluctance network method.

II. Equations of Magnetic Field Distribution

In order to formulate the equations of the magnetic field distribution, the appropriate components of field intensities and components of flux densities are assigned to individual segments, as it is shown in Fig. 2. The *H* components have the *m* subscript if they do not belong to the network tree, and the remaining components are depicted by the *p* subscript. The flux density components which are assigned to the left lower subsegments have the *b* subscripts, and the *t* subscript depicts the components associated with the right upper subsegments. On the basis of the first Maxwell equation it is possible to formulate algebraic equations for independent meshes in the following form

$$\sum_{k=1}^{4} a_k H_k = s_J J \tag{1}$$

where H_k is a field intensity component, a_k denotes the distance between the corresponding vertices of the segment, J presents density of an external current, and s_J denotes the area of the surface determined by a mesh.

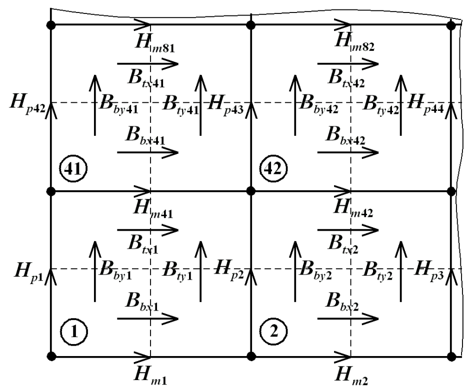

Fig. 2. An example of the description of the field intensity and flux density components

All equations can be written in the following matrix form

$$\boldsymbol{A_m H_m} + \boldsymbol{A_p H_p} = \boldsymbol{S_J J_{ex}} \tag{2}$$

where $\boldsymbol{H_m}$, $\boldsymbol{H_p}$ are the column vectors of the components H_m, H_p respectively, $\boldsymbol{A_m}$, $\boldsymbol{A_p}$ denote the square matrixes of the distances a_k, with which the components H_m or H_p are associated, $\boldsymbol{S_J}$ is the matrix of areas s_J, and $\boldsymbol{J_{ex}}$ denotes the column vector of the density values of external currents.

Using the Gauss law for magnetic field in the integral form it is possible to write algebraic equations for independent nodes of the network (Fig. 2) as follows

$$\sum_{k=1}^{ls} c_k B_k = 0 \text{ or } \Phi_{ex} \tag{3}$$

where B_k is a flux density component, c_k denotes the area of the segment face which the magnetic flux with the flux density component B_k penetrates, l_s is the number of the magnetic flux density components associated with the given node, and Φ_{ex} denotes a certain external magnetic flux which can flow into the given node.

All equations in the form (3) can be written in the matrix form

$$\boldsymbol{C_{bx}B_{bx}} + \boldsymbol{C_{by}B_{by}} + \boldsymbol{C_{tx}B_{tx}} + \boldsymbol{C_{ty}B_{ty}} = \boldsymbol{\Phi_{ex}} \tag{4}$$

where $\boldsymbol{B_{bx}}$, $\boldsymbol{B_{by}}$, $\boldsymbol{B_{tx}}$, and $\boldsymbol{B_{ty}}$ are column vectors of components B_{bx}, B_{by}, B_{tx}, and B_{ty} respectively, $\boldsymbol{C_{bx}}$, $\boldsymbol{C_{by}}$, $\boldsymbol{C_{tx}}$, $\boldsymbol{C_{ty}}$ denote matrixes of segment face areas which are penetrated by magnetic fluxes with the corresponding components, and $\boldsymbol{\Phi_{ex}}$ is the vector column of external fluxes.

III. Inclusion of the Rotational Magnetization Model

The field intensities of the so-called direction hystereses depend on the H_m and H_p components and on the angle between directions specified on the steel sheet plane and the x and y axes of the coordinate system (Fig. 1). For example, the field intensity for the chosen direction numbered 4 (Fig. 1) in segment 42 (Fig. 2) is equal to

$$H_{42d4} = \cos 3\alpha H_{m41} + \cos \alpha H_{p43} \tag{5}$$

where α is the angle between two neighbouring directions.

The flux density components B_{bx}, B_{by}, B_{tx}, and B_{ty} are equal to the sums of the projections on the x-axis and y-axis of flux densities in individual directions. For instance, the components B_{bx}, B_{by} in segment 42 can be written as follows

$$B_{42bx} = B_{42d1} + \cos\alpha B_{42d2} + \ldots + \cos 7\alpha B_{42d8} \tag{6}$$

$$B_{42by} = \cos 3\alpha B_{42d2} + \cos 2\alpha B_{42d3} + \ldots + \cos 3\alpha B_{42d8} \tag{7}$$

where B_{ndk} are the flux densities of the direction hystereses.

The flux densities of direction hysteresis are nonlinear functions of the field intensities in individual directions, i.e. $B_{kd1}=f(H_{kd1})$, $B_{kd2}=f(H_{kd2})$, $B_{kd3}=f(H_{kd3})$, and so on. As a result, the B_{bx}, B_{by}, B_{tx}, and B_{ty} components are functions of the H_m and H_p components. All flux density components are arranged into appropriate column vectors. This allows us to transform (4) to the form in which the column vectors of the H_m and H_p components are unknown. It results from (2) that the $\boldsymbol{H_m}$ column vector is dependent on the $\boldsymbol{H_p}$ vector. Then, only the $\boldsymbol{H_p}$ vector is the unknown vector in the modified (4). The final equation of the magnetic field distribution is nonlinear and this equation is solved with the use of the Newton-Raphson method. It is worth underlining that due to the variable magnetic field, the calculations of the magnetic field distribution should be carried out with taking eddy currents into account [5]. This is achieved by taking into account a separate network and additional equations for these currents.

IV. Experimental Verification

The experimental verification was performed for chosen non-oriented steel sheets. The alternating magnetic field was induced in the pack of electrically insulated steel sheets by two mutually perpendicular coils (Fig. 3). For different currents, the voltages of the measurement coils were stored.

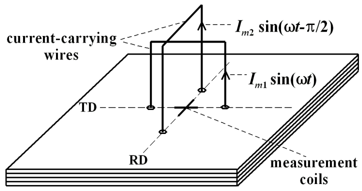

Fig. 3. Measurement system; RD, TD – rolling, transverse direction respectively

These voltages were compared with the corresponding voltages calculated numerically for the same conditions, as in the experiment. Figure 4 shows the waveforms of these voltages stored and calculated for the non-oriented steel sheet M400-50 A produced in Sweden (thickness 0.5 mm) in the rotational and alternating magnetization. The presented waveforms relate to the rolling direction (RD).

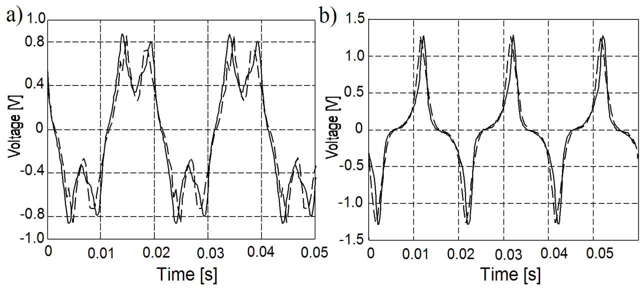

Fig. 4. Voltages of the measurement coil in the rolling direction:
a) rotational magnetization, b) alternating magnetization; continuous line – measured waveform, dashed line – calculated waveform.

Acknowledgments

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