FEM technique for modelling eddy current testing of ferromagnetic media with low skin depth

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***Abstract*—In this paper, an approach has been used for modeling a massive domain in the eddy current testing applications. Some materials can have an important relative permeability inducing a low skin thickness. With the purpose of modeling, the classical finite element method can become unsuitable. The overlapping finite element method is consequently implemented for this purpose.**

***Index Terms*— Finite element method, non-destructive testing, eddy currents, ferromagnetic materials, thin layers.**

I. Introduction

The non-destructive testing by means of the eddy current technique (ECT) is nowadays frequently used to evaluate the properties of conducting materials. The interpretation of the signal variations received by the probe (due for instance to a crack) involves the use of mathematical models. These models permit also to study configurations that cannot be reproduced without difficulty experimentally.

The finite element method is often chosen due to its high adaptability when using simplex elements (triangles in 2D or tetrahedra in 3D). However a correct modelling of ECT requires some rules: accurate meshes for the probe and the surrounding air, fine mesh in the lift-off (distance between the probe and the conducting part) and usually considering first order elements at least three layers of finite elements have to be used in the skin-depth. This condition becomes critical when considering ECT of ferromagnetic materials having a low skin depth. Indeed the meshing of these materials will result either in highly distorted simplex finite elements or in a high density mesh leading to a huge number of degrees of freedoms (DOF). This paper proposes an efficient technique for modelling ECT of ferromagnetic materials based on the overlapping method. In previous works, numerical techniques based on this method have been proposed in order to take into account a thin geometrical domain thanks to a layer of overlapping elements [1][2]. In this paper we propose to apply this technique in order to model efficiently the skin effect in low skin depth domains.

The two first parts of this paper describe briefly the used finite element formulations and the overlapping finite element method. In the second part some explanations are given about the proposed approach to take into account the skin effect. Finally, a test case is considered and computed results are compared to analytical and experimental data.

II. Finite element formulations

There are two families of formulations for solving Maxwell's equations. One is based on the calculation of the electric field $\boldsymbol{e}$ and the other on the calculation of the magnetic field $\boldsymbol{h}$ [3]. The magnetic field $\boldsymbol{h}$ is described as a function of the vector potentials $\boldsymbol{t}$ and $\boldsymbol{t_0}$ and with the help of the scalar potential ϕ:

$$\boldsymbol{h} = \boldsymbol{t} + \boldsymbol{t_0} - \boldsymbol{grad}\,\phi \qquad (1)$$

The vector potentials $\boldsymbol{t}$ and $\boldsymbol{t_0}$ come from the relations $\boldsymbol{curl}\ \boldsymbol{t} = \boldsymbol{j}$ and $\boldsymbol{curl}\ \boldsymbol{t} = \boldsymbol{j_0}$, where $\boldsymbol{j}$ is the density of eddy currents and $\boldsymbol{j_0}$ is the density of source current. With the introduction of a magnetic vector potential $\boldsymbol{a}$ and an electric scalar potential ψ, the electrical field $\boldsymbol{e}$ is expressed with (2).

$$\boldsymbol{e} = -\partial_t(\mathbf{a} + \boldsymbol{grad}\ \psi) \qquad (2)$$

The potential $\boldsymbol{a}$ comes from the relation $\boldsymbol{b}=\boldsymbol{curl}\ \boldsymbol{a}$ where $\boldsymbol{b}$ is the magnetic flux density. The fields and potentials are associated to the Whitney elements. The 3D geometry is discretized using tetrahedrons. The modeling is made in harmonic regime. The obtained algebraic system is solved using the conjugate gradient method.

III. Overlapping Finite element method

The overlapping finite element method has been firstly introduced in [4] in 2D and applied recently in 3D [2] for ECT applications. Its principle is to connect two domains separated by an initially unmeshed region (see Fig.1a.). The shape functions belonging to the nodes i and j are defined only in D_1 and D_2, respectively. A projection of the nodes of D_1 is performed in D_0 up to the boundary of D_2 and reciprocally from D_2 toward D_1 for the D_2 nodes, see Fig.1(b).

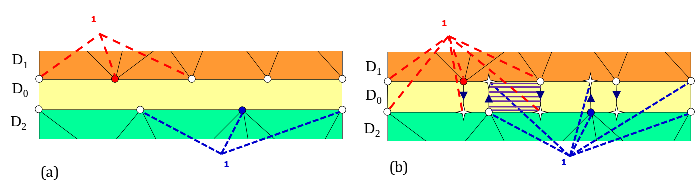

Fig. 1. (a) : Two domains independently meshed, (b): nodal projections.

New shape functions are introduced in D_0. These functions are represented in Fig.1(b) by quadrilateral elements (or prisms in 3D) that overlap in D_0. In this region, integral terms are computed in the area where shape functions are commonly nonzero (see the hatched area in Fig.1(b) related to the nodes i and j) and added to the matrix system. The real nodes (designed by a star) are not taken as unknown but, depending on the used formulation, new vertical edges have to be added in the problem.

IV. Modelling of the Skin Effect

In ferromagnetic material ECT the skin depth is usually small due to the high value of the permeability. The eddy current density is important in the skin depth and decreases quickly and approximately exponentially. A fine mesh is then required on the periphery of the material .The idea is then to superpose several overlapping finite element layers in the first skin depths of the ferromagnetic domain. With the purpose to limit the number of DOF, the thickness of the layers varies exponentially with the depth z, see Fig.2.

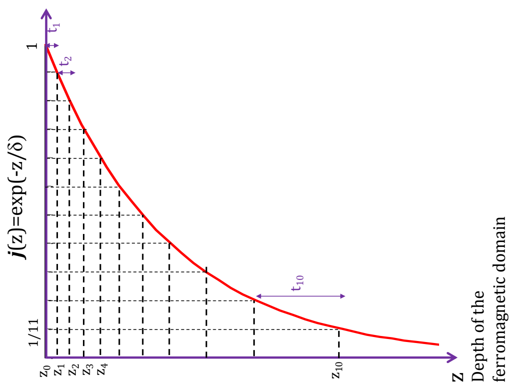

Fig. 2. Discretization of the first skin depths of the ferromagnetic domain in several layers of overlapping elements

V. Test Case and Comparisons

The studied problem is a simple coil above a ferromagnetic plate without defect. It comes from a configuration proposed in [5] with associated experimental and analytical results. The main parameters of the problem are given in Table I.

TABLE I
Parameters of the test case [5]

External radius of the coil	10.7 mm
Internal radius of the coil	6.88 mm
Height of the coil	5 mm
Number of turns of the coil	410
Thickness of the lift-off	1.9 mm
Thickness of the plate	10 mm
Conductivity of the plate	3.47 MS/m
Relative permeability of the plate	85

The working frequency range is very large: from 0.1 kHz to 100 MHz. Consequently the skin depth in the plate varies considerably, from 2.9 mm to 2.9 μm. It was chosen to work with 10 overlaping layers whose thicknesses are equal to ($1 \leq i \leq 10$):

$$t_i = (z_i - z_{i-1}) \quad \text{with} \quad z_i = \ln(11/11-i)\,\delta \qquad (3)$$

where δ is the skin depth of the plate. By this way, 2.39 δ of the plate depth are discretized with the overlapping method. The rest of the plate (inside) is meshed with tetrahedral elements. For the lowest frequencies (≤ 10 kHz) the thicknesses of the overlapping layers have been kept constant in order to avoid elements of too large thickness. For these low frequencies the classical finite element method can be used. In the considered problem the thickness of lift-off is sufficiently important to be meshed by tetrahedral elements.

The proposed approach has been implemented and the average value calculated from the two formulations of the normalized resistance and reactance of the coil have been computed.The results are given in Fig.3 in function of the frequency and compared to analytical and experimental data issued from [5].

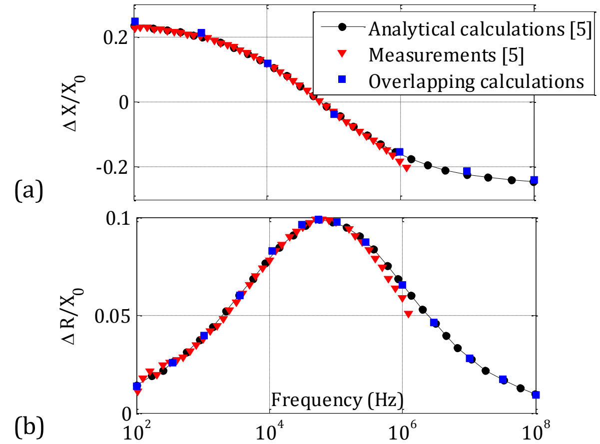

Fig. 3. (a) : Normalized reactance in function of the frequency,
(b) : Normalized resistance in function of the frequency

It can be noticed that the results are in good agreement. The method allows extending significantly the frequency range of application of the FEM by comparison to classical FEM. Limitations of classical FEM (and of other numerical techniques) can be found in [6] with the same test case under consideration.

VI. Conclusion

It can be concluded from this test case that the overlapping method can take into account correctly ferromagnetic medium in high-frequency and more generally stratified media.

Acknowledgements

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