Analysis of the Motion of Conducting Sheets in Magnetic Fields

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***Abstract*—Solution of the equation of motion for conductors in external magnetic fields requires the knowledge of the magnetic forces due to the induced eddy currents that, in turn, can be determined if the position and the velocity of the bodies are known. An iterative technique is adopted, where, at each time step, an initial value of the magnetic force is used to determine the position and the velocity of the body at the end of the time step and, then, the value of the force is corrected. To reduce the computational effort in the case of thin metallic sheets, it is proposed to use the surface integral equation of the induced eddy currents, with a supplementary term added to account for the motion. A sinusoidal with time variation of the excitation field is considered and a phasor representation of various physical quantities is employed. For sufficiently high frequencies, an average magnetic force over each period of time is used, which varies as the position of the moving sheet changes.**

***Index Terms*—Computational electromagnetics, eddy currents, electromagnetic forces, integral equations, magnetic levitation.**

I. Introduction

The space evolution of a conducting body in the presence of external magnetic fields can be determined by solving the equation of motion taking into account the magnetic forces due to the induced eddy currents. To compute the magnetic forces, one has to solve the electromagnetic field problem, with the position and the velocity of the body not known, which have also to be determined at each stage. This mechanical-electromagnetic coupling is treated by using an iterative procedure, where, at each time step chosen for discretizing the motion equation, the electromagnetic field and the magnetic force and, then, the position and the velocity of the body are updated. Application of the finite element method requires a huge computational effort, at each iteration step being necessary to generate a new discretization grid. The hybrid finite element – boundary element method [1] is much more efficient, the finite element grid being only associated with the conducting body and, thus, remaining unchanged. The eddy-current integral equation in [2] provides a particularly efficient procedure that can be extended to nonlinear media [3].

To increase the magnetic forces, induced currents of higher intensity are needed, which can be generated by employing excitation fields of sufficiently high frequencies. The time-domain solution of the eddy-current problem and of the equation of motion would require to choose a time step smaller than the period of excitation. The corresponding amount of computation is huge. Since the "mechanical" time constant is much greater than the "electrical" one, in [4] a procedure has been proposed, where the equation of motion is solved in time domain choosing a time step which is much greater than the period of the excitation field, while the eddy-current problem is treated using a phasor representation. A supplementary term is introduced in the eddy-current integral equation to account for the conductor motion. The procedure has been developed and implemented only for two-dimensional structures, the necessary computational time being spectacularly decreased.

In this work, we present a technique for determining the motion of the three-dimensional thin conducting sheets under the action of magnetic forces due to the induced eddy currents. The surface integral equation satisfied by the eddy currents, with a "motional" term depending on the relative velocity of the conducting sheet, is solved numerically, the number of unknowns being given by the number of nodes in the sheet discretization mesh.

II. Current Sheet Integral Equation

For a thin conducting sheet S, the current density is assumed to have a surface distribution. In the system of coordinates attached to the sheet, the integral equation of the surface current density is

$$\frac{1}{\sigma_S}\boldsymbol{J}_S(\boldsymbol{r},t)+\frac{\mu_0}{4\pi}\frac{d}{dt}\int_S\frac{\boldsymbol{J}_S(\boldsymbol{r}',t)}{R}dS'=\boldsymbol{E}_0(\boldsymbol{r},t)-\nabla\Phi\,,\ \boldsymbol{r}\in S \quad (1)$$

where $\sigma_S=\sigma\Delta$ is the sheet surface conductivity, with Δ the sheet thickness, μ_0 is the permeability of free space, $R=|\boldsymbol{r}-\boldsymbol{r}'|$ with $\boldsymbol{r}$ and $\boldsymbol{r}'$, respectively, the position vectors of the observation point and of the source point, $\boldsymbol{E}_0$ is the electric field intensity due to the external moving sources, and $-\nabla\Phi$ is the scalar potential component of the electric field intensity. Denoting by $\boldsymbol{J}_i$ the given current density within the volume of the excitation coils, we have

$$\boldsymbol{E}_0(\boldsymbol{r},t)=-\frac{\mu_0}{4\pi}\frac{d}{dt}\int_{\Omega_i}\frac{\boldsymbol{J}_i(\boldsymbol{r}',t)}{R}dV'$$

$$=-\frac{\mu_0}{4\pi}\int_{\Omega_i}\frac{1}{R}\frac{\partial\boldsymbol{J}_i(\boldsymbol{r}',t)}{\partial t}dV'-\frac{\mu_0}{4\pi}\int_{\partial\Omega_i}\frac{\boldsymbol{J}_i(\boldsymbol{r}',t)}{R}(\boldsymbol{v}\cdot\boldsymbol{n})dS' \quad (2)$$

where Ω_i is the region occupied by the coils, $\partial\Omega_i$ is the boundary of Ω_i, $\boldsymbol{n}$ is the outwardly oriented normal unit vector of $\partial\Omega_i$, and $\boldsymbol{v}$ is the velocity of the excitation coils in the frame of reference attached to the conducting sheet S. The last term in (2) is an additional term due to the relative motion between the excitation coils and the induced conducting sheet.

III. Numerical Solution

Since the frequency of the excitation fields considered is much greater than the frequency of the sheet mechanical oscillations, we assume that the geometry of the system remains practically unchanged during each period of the excitation fields and, therefore, (1) can be solved by using a phasor representation. The surface of the conducting sheet is modeled as a polyhedral surface with triangular surface elements and the current density is approximated by a linear combination of vector functions $\boldsymbol{U}_i$ associated with each interior node (i) of the surface as [5]

$$\boldsymbol{J}_s(\boldsymbol{r}) \approx \sum_{i=1}^{N} \alpha_i \boldsymbol{U}_i(\boldsymbol{r}) \tag{3}$$

where $\boldsymbol{U}_i$ has a constant value, $\boldsymbol{U}_i^{(p)} = \dfrac{1}{2S_p}\boldsymbol{l}_i^{(p)}$, over each surface element *(p)*, of area S_p, containing the node (i), and a zero value for all the surface elements which do not contain the node (i), $\boldsymbol{l}_i^{(p)}$ being the length vector along the edge of the element (p) that is opposed to the node (i). Taking the scalar product of the terms in (1) with $\boldsymbol{U}_n$, $n = 1,2,\ldots,N$, where N is the number of interior nodes, yields the system of algebraic equations

$$\sum_{i=1}^{N} A_{ni}\alpha_i = C_n, \quad n = 1,2,\ldots,N \tag{4}$$

with

$$A_{ni} = \int_S \frac{1}{\sigma_s}\boldsymbol{U}_n \cdot \boldsymbol{U}_i dS + \frac{j\mu_0 f}{2}\int_S\int_S \frac{1}{R}\boldsymbol{U}_n \cdot \boldsymbol{U}_i dS' dS \tag{5}$$

$$C_n = \int_S \boldsymbol{U}_n \cdot \boldsymbol{E}_0 dS \tag{6}$$

where $j \equiv \sqrt{-1}$ and f is the frequency of the coil currents. The term $-\nabla\Phi$ in (1) does not contribute to the system (4) [5].

IV. Magnetic Force

Using the solution of the system of equations (4), the average magnetic force over a period of the excitation field is computed as

$$\boldsymbol{F} = \mathrm{Re}\left[\sum_{k=1}^{N_f}\left(\boldsymbol{J}_{sk}^* \times \boldsymbol{B}_{ik}\right)S_k\right] \tag{7}$$

where $\boldsymbol{B}_{ik}$ is the average magnetic induction over the surface element (k), produced by the excitation coil currents, N_f is the number of surface elements of S, and * indicates the complex conjugate. When the linear dimensions of the surface elements on S are sufficiently small with respect to the distances to the excitation coils, $\boldsymbol{B}_{ik}$ can approximately be taken to be the value of the magnetic induction at the center of the respective surface element. This average force varies with time, from one period of the excitation current to the next one.

V. Equation of Motion

The classical mechanics equation is used,

$$m\frac{d^2\boldsymbol{r}}{dt^2} = \boldsymbol{F}(\boldsymbol{r}, \frac{d\boldsymbol{r}}{dt}, \boldsymbol{J}_i) + \boldsymbol{G} \tag{8}$$

where m is the mass of the sheet and $\boldsymbol{G}$ is its weight. $\boldsymbol{F}$ is calculated with (7) and depends on the sheet position and velocity, and on the value of the excitation current density. Equation (8) is solved iteratively, following the scheme in [4]. A time step which can be much greater than the period of the excitation current is chosen and the force is assumed to vary linearly within the time step. Using an initial value of the force, the new position and velocity of the sheet are determined at the end of the time step. Then, the value of the force is corrected with (7), and (8) is solved again. When the difference between successive values at the end of the time step is sufficiently small, one passes to the next time step.

VI. Conclusions

An efficient procedure is presented for the determination of the motion of a three-dimensional conducting sheet in the presence of sinusoidal with time external magnetic fields. Taking into account that the space evolution of the sheet is much slower than the evolution of the electromagnetic field quantities, it is proposed to integrate the equation of motion using the average value of the magnetic force over a period of the external field. The classical surface integral equation of the eddy currents is modified by adding a term in order to take into account the motion of the sheet.

Illustrative computation examples will be presented in the long version of the paper.

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